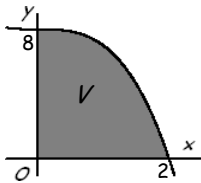


1 De tweede benadering is de beste.

2 Omdat de grafiek van f dalend is, is het minimum van f op elk interval de functiewaarde in de rechtergrens van het interval. En zo is het maximum van f op elk interval de functiewaarde in de linkergrens van het interval.

3 $y=0$ (de x -as)
 $f(x)=0$
 $8-x^3=0$
 $x^3=8$
 $x=2.$



De middens van de intervallen zijn 0,2; 0,6; 1; 1,4 en 1,8.
 $O(V) \approx f(0,2) \cdot 0,4 + f(0,6) \cdot 0,4 + f(1) \cdot 0,4 + f(1,4) \cdot 0,4 + f(1,8) \cdot 0,4$
 $= (f(0,2) + f(0,6) + f(1) + f(1,4) + f(1,8)) \cdot 0,4 = 12,08.$


Plot1 Plot2 Plot3
 $\sqrt{y} = 8 - x^3$
 $\sqrt{y} =$

Plot1 Plot2 Plot3
 $V_1(0,2)+V_1(0,6)+V_1(1,8)$
 $\text{Ans} \approx 0,4 \quad 30,2$
 $\quad \quad \quad 12,08$

4 $f(x)=0$
 $\sqrt{2x+6}=0$
 $2x+6=0$
 $2x=-6$
 $x=-3.$

Plot1 Plot2 Plot3
 $\sqrt{y} = \sqrt{2x+6}$
 $\sqrt{y} =$
 $\sqrt{y} =$

WINDOW
 $X_{\min}=-3$
 $X_{\max}=0$
 $X_{\text{scl}}=0$
 $Y_{\min}=0$
 $Y_{\max}=4$
 $Y_{\text{scl}}=0$
 $X_{\text{res}}=1$



$V_1(-3)+V_1(-2,5)+V_1(-2)+V_1(-1,5)+V_1(-1)+V_1(-0,5)$
 $\text{Ans} \approx 0,5 \quad 4,191166174$

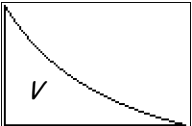
$V_1(-2,5)+V_1(-2)+V_1(-1,5)+V_1(-1)+V_1(-0,5)+V_1(0)$
 $\text{Ans} \approx 0,5 \quad 5,415911045$

ondersom = $(f(-3) + f(-2,5) + f(-2) + f(-1,5) + f(-1) + f(-0,5)) \times 0,5 \approx 4,19.$
 bovensom = $(f(-2,5) + f(-2) + f(-1,5) + f(-1) + f(-0,5) + f(0)) \times 0,5 \approx 5,42.$ Dus $4,19 \leq O(V) \leq 5,42.$

5a $y=0$ (de x -as)
 $f(x)=0$
 $\frac{12-2x}{x+4}=0$ (teller = 0)
 $12-2x=0$
 $x=6.$

Plot1 Plot2 Plot3
 $\sqrt{y} = \frac{12-2x}{x+4}$
 $\sqrt{y} =$
 $\sqrt{y} =$
 $\sqrt{y} =$
 $\sqrt{y} =$
 $\sqrt{y} =$

WINDOW
 $X_{\min}=0$
 $X_{\max}=6$
 $X_{\text{scl}}=0$
 $Y_{\min}=0$
 $Y_{\max}=3$
 $Y_{\text{scl}}=0$
 $X_{\text{res}}=1$



$V_1(0,5)+V_1(1,5)+V_1(2,5)+V_1(3,5)+V_1(4,5)+V_1(5,5)$
 $\text{Ans} \approx 1 \quad 6,282602159$

$V_1(1)+V_1(2)+V_1(3)+V_1(4)+V_1(5)+V_1(6)$
 $\text{Ans} \approx 1 \quad 4,912698413$

De middens van de intervallen zijn 0,5; 1,5; 2,5; 3,5; 4,5 en 5,5.
 $O(V) \approx (f(0,5) + f(1,5) + f(2,5) + f(3,5) + f(4,5) + f(5,5)) \cdot 1 \approx 6,28.$

5b ondersom = $(f(1) + f(2) + f(3) + f(4) + f(5) + f(6)) \cdot 1 \approx 4,91.$
 bovensom = $(f(0) + f(1) + f(2) + f(3) + f(4) + f(5)) \cdot 1 \approx 7,91.$ Dus $4,91 \leq O(V) \leq 7,91.$

6a De oppervlakte van de blauwe rechthoek rechts wordt gehalveerd.
 6b De oppervlakte van de blauwe rechthoek rechts is het verschil van de ondersom en de bovensom. De breedte van deze rechthoek is Δx . Als $\Delta x \rightarrow 0$ dan nadert de oppervlakte van deze blauwe rechthoek naar 0 en nadert het verschil van de ondersom en de bovensom dus ook naar 0.

*** **Neem GR - practicum 10 door.** (zie aan het eind van deze uitwerkingen)

7 $f(x)=3 \Rightarrow \sqrt{x}=3$ (kwadrateren) $\Rightarrow x=9.$

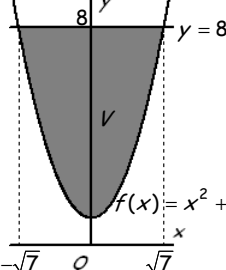
NUM CPX PRB
 $3 \cdot 9 - \text{fnInt}(f(x), x, 0, 9)$
 $\text{Ans} \approx 8,999999749$



$O(V) = 3 \cdot 9 - \int_0^9 \sqrt{x} dx$ (fnInt) $\approx 9,00.$

8 $f(x)=8 \Rightarrow x^2+1=8$
 $x^2=7$
 $x=\pm\sqrt{7}.$

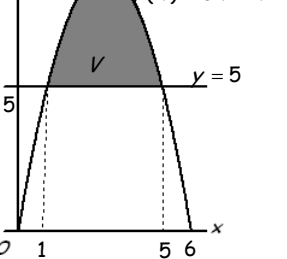
$2 \cdot \sqrt{7} \cdot 8 - \text{fnInt}(x^2+1, x, -\sqrt{7}, \sqrt{7})$
 $\text{Ans} \approx 24,6936789$



$O(V) = 2 \cdot \sqrt{7} \cdot 8 - \int_{-\sqrt{7}}^{\sqrt{7}} (x^2+1) dx$ (fnInt) $\approx 24,69.$

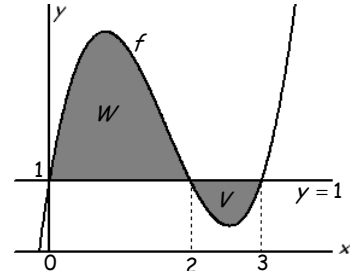
9 $f(x)=5 \Rightarrow 6x-x^2=5$
 $-x^2+6x-5=0$
 $x^2-6x+5=0$
 $(x-1) \cdot (x-5)=0$
 $x=1 \vee x=5.$

$\text{fnInt}(6x-x^2, x, 1, 5) - 4 \cdot 5$
 $\text{Ans} \approx 10,66666667$



$O(V) = \int_1^5 (6x-x^2) dx$ (fnInt) $- 4 \cdot 5 \approx 10,67.$

10a $f(x)=1 \Rightarrow x^3 - 5x^2 + 6x + 1 = 1$
 $x^3 - 5x^2 + 6x = 0$
 $x \cdot (x^2 - 5x + 6) = 0$
 $x \cdot (x-2) \cdot (x-3) = 0$
 $x = 0 \vee x = 2 \vee x = 3.$



$O(V) = 1 \cdot 1 - \int_2^3 (x^3 - 5x^2 + 6x + 1) dx$ (fnInt) $\approx 0,42.$

10b $O(W) = \int_0^2 (x^3 - 5x^2 + 6x + 1) dx$ (fnInt) $- 2 \cdot 1 \approx 2,67.$

11a $O(U) = \int_1^8 f(x) dx = \int_1^8 (10x - x^2) dx$ (fnInt) $\approx 144,67$ en $O(V) = \int_1^8 g(x) dx = \int_1^8 (x + 8) dx$ (fnInt) $= 87,5.$

11b $O(W) = O(U) - O(V) = \int_1^8 f(x) dx - \int_1^8 g(x) dx$ (zie 11a) $\approx 57,17.$

11c $\int_1^8 f(x) dx - \int_1^8 g(x) dx = \int_1^8 (f(x) - g(x)) dx.$ (zie figuur 10.5)

fnInt(10X-X^2,X,1,8)
 +U
 144,6666667
 fnInt(X+8,X,1,8)
 +V
 87,5
 U-V
 57,16666667
 fnInt(10X-X^2-(X+8),X,1,8)
 57,16666667

*** **Neem GR-practicum 11 door.** (zie aan het eind van deze uitwerkingen)

12a $f(x) = g(x)$ (intersect) $\Rightarrow x = 0$ (lukt niet met intersect) $\vee x = 1.$
 Op $[0,1]$ (zie plot) is $g(x) \geq f(x)$, dus

$O(V) = \int_0^1 (g(x) - f(x)) dx$ (fnInt) $\approx 0,33.$

WINDOW
 Xmin=-1
 Xmax=5
 Xscl=1
 Ymin=-2
 Ymax=1
 Yscl=1
 Xres=1

Plot1 Plot2 Plot3
 V1: X-2f(X)
 V2: -X
 V3: =
 V4: =

Intersection
 N=1
 X=1
 Y=1

fnInt(V2-V1,X,0,1)
 .3333337539

12b $f(x) = 0$ (intersect) $\Rightarrow x = 0$ (lukt niet met intersect) $\vee x = 4.$
 Op $[0,4]$ (zie plot) is $f(x) \leq 0$, dus

$O(V) = \int_0^4 (0 - f(x)) dx$ (fnInt) $\approx 2,67.$

WINDOW
 Xmin=-1
 Xmax=5
 Xscl=1
 Ymin=-2
 Ymax=1
 Yscl=1
 Xres=1

Plot1 Plot2 Plot3
 V1: X-2f(X)
 V2: 0
 V3: =
 V4: =

Intersection
 N=4
 X=4
 Y=0

fnInt(0-V1,X,0,4)
 2,66667087

13 $f(x) = g(x)$ (intersect) $\Rightarrow x \approx -1,32 \vee x \approx -0,43 \vee x \approx 1,75.$

$O(V) = \int_{-1,32}^{-0,43} (f(x) - g(x)) dx + \int_{-0,43}^{1,75} (g(x) - f(x)) dx$ (fnInt) $\approx 3,75.$

WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-2
 Ymax=2
 Yscl=1
 Xres=1

Plot1 Plot2 Plot3
 V1: X^3-3X
 V2: 1-1/2X
 V3: =
 V4: =

fnInt(V1-V2,X,A,B)+fnInt(V2-V1,X,B,C)
 3,747281687

Intersection
 N=1.320012
 Y=1.6600059

X+A -1.320011733

Intersection
 N=-4.323204
 Y=1.2161602

X+B -1.320011733

X+C -4.323204433

Intersection
 N=1.7523322
 Y=1.2383381

X+A -1.320011733

X+B -4.323204433

X+C 1.752332177

14 $f(x) = g(x)$ (intersect) $\Rightarrow x \approx 2,47.$

$\int_0^\pi f(x) dx$ (fnInt) $= 2$ en

$O(V) = \int_0^{2,47} (f(x) - g(x)) dx$ (fnInt) $\approx 1,02.$

Het is dus niet waar want $1,02 \neq \frac{1}{2} \cdot 2.$

Plot1 Plot2 Plot3
 V1: sin(X)
 V2: 1/4X
 V3: =
 V4: =
 V5: =
 V6: =
 V7: =

WINDOW
 Xmin=0
 Xmax=pi
 Xscl=0
 Ymin=-.5
 Ymax=1.5
 Yscl=0
 Xres=1

Intersection
 N=2.4745768
 Y=.6186442

X+A 2.474576787

X+A 2.474576787
 fnInt(V1,X,0,pi)
 2
 fnInt(V1-V2,X,0,A)
 1.020229994

15a $O(V) = \int_0^\pi \sin(x) dx$ (fnInt) $= 2$ en $O(W) = \int_0^{2\pi} (0 - \sin(x)) dx$ (fnInt) $= 2.$

15b $\int_0^{2\pi} \sin(x) dx$ (fnInt) $= 0.$ $\int_0^{2\pi} \sin(x) dx = \int_0^\pi \sin(x) dx + \int_\pi^{2\pi} \sin(x) dx = 2 + (-2) = 0.$

De integraal is negatief als de grafiek onder de x-as ligt.

15c $\int_0^{2\pi} |\sin(x)| dx$ (fnInt) $= 4.$ Dus $O(V) + O(W) = \int_0^{2\pi} |\sin(x)| dx.$

Plot1 Plot2 Plot3
 V1: sin(X)
 V2: =
 V3: =

WINDOW
 Xmin=0
 Xmax=2pi
 Xscl=0
 Ymin=-.5
 Ymax=1.5
 Yscl=1
 Xres=1

fnInt(V1,X,0,pi)
 2
 fnInt(0-V1,X,pi,2pi)
 2

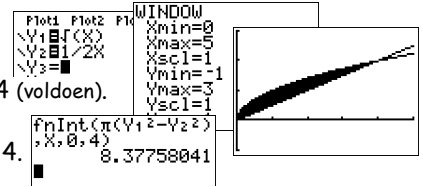
fnInt(V1,X,0,2pi)
 0

MATH NUM CPX PRB
 1: abs(
 2: round
 3: iPart
 4: fPart
 5: int(
 6: min(
 7: max(
 Plot1 Plot2 Plot3
 V1: abs(sin(X))
 V2: =
 V3: =

- 16 $f(x) = 0$ (intersect of) $\Rightarrow x \cdot (x^2 - 5x + 6) = 0 \Rightarrow x \cdot (x - 2) \cdot (x - 3) = 0 \Rightarrow x = 0 \vee x = 2 \vee x = 3$.
 $O(V) + O(W) = \int_0^3 |f(x)| dx$ (fnInt) $\approx 3,08$.
- 17 $O = \int_{-1,32}^{1,75} |f(x) - g(x)| dx$ (fnInt) $\approx 3,75$.
 (vanwege de absolute waarde maakt het niet uit of $f(x) > g(x)$ of $f(x) < g(x)$)
- 18 $f(x) = g(x)$ (intersect) $\Rightarrow x \approx -1,96 \vee \dots \vee x \approx 1,83$.
 (alleen de buitenste snijpunten zijn van belang)
 $O(V) + O(W) = \int_{-1,96}^{1,83} |f(x) - g(x)| dx$ (fnInt) $\approx 5,48$.
- 19a Op $[0, 2]$: $I_{cilinder} = \pi \cdot r^2 \cdot h = \pi \cdot f(1)^2 \cdot 2 = \pi \cdot 11^2 \cdot 2 = 242\pi$.
 Op $[2, 4]$: $I_{cilinder} = \pi \cdot r^2 \cdot h = \pi \cdot f(3)^2 \cdot 2 = \pi \cdot 19^2 \cdot 2 = 722\pi$.
 Op $[4, 6]$: $I_{cilinder} = \pi \cdot r^2 \cdot h = \pi \cdot f(5)^2 \cdot 2 = \pi \cdot 35^2 \cdot 2 = 2450\pi$.
- 19b I_3 cilinders $= 242\pi + 722\pi + 2450\pi = 3414\pi \approx 10725$.
- 20 $f(x) = 0$ (intersect of) $\Rightarrow 8 - 2^x = 0 \Rightarrow 2^x = 8 = 2^3 \Rightarrow x = 3$.
 $I(L) = \int_0^3 \pi \cdot (f(x))^2 dx$ (fnInt) $\approx 238,33$.
- 21 $f(x) = 0$ (intersect) $\Rightarrow x = -3,14 \vee x = 3,83$.
 $I(L) = \int_{-3,14}^{3,83} \pi \cdot (f(x))^2 dx$ (fnInt) $\approx 487,49$.
- 22 $f(x) = 2x + 6$ (intersect of) $\Rightarrow 9 - x^2 = 2x + 6 \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow (x + 3) \cdot (x - 1) = 0 \Rightarrow x = -3 \vee x = 1$.
 $I(L) = \int_{-3}^1 \pi \cdot (2x + 6)^2 dx + \int_1^3 \pi \cdot (f(x))^2 dx$ (fnInt) $\approx 438,99$.
- 23a $g(x) = f(x) - 2 = 2 + \frac{3}{x} - 2 = \frac{3}{x}$.
- 23b $I(M) = \int_1^3 \pi \cdot (g(x))^2 dx$ (fnInt) $\approx 18,85$.
 Wentelen van V om de lijn $y = 2$ levert een lichaam met dezelfde inhoud op als wentelen van W om de x -as, omdat V en de lijn $y = 2$ beide 2 omlaag zijn verschoven.
- 24a $I = \int_1^4 \pi \cdot (f(x))^2 dx - \int_1^4 \pi \cdot 1^2 dx$ (fnInt) $\approx 11,07$.
- 24b $I = \int_1^4 \pi \cdot (f(x) - 1)^2 dx$ (fnInt) $\approx 2,36$.
- 25a $I(L) = \int_0^8 \pi \cdot (\frac{1}{2}x)^2 dx$ (fnInt) $\approx 134,04$.
- 25b $I(M) = \int_0^8 \pi \cdot 4^2 dx - I(L)$ (fnInt) $= \pi \cdot 4^2 \cdot 8 - I(L)$ (fnInt) $\approx 268,08$.
- 26 $I(L) = \pi \cdot 2^2 \cdot 5 - \pi \cdot 1^2 \cdot 5 = 20\pi - 5\pi = 15\pi$.

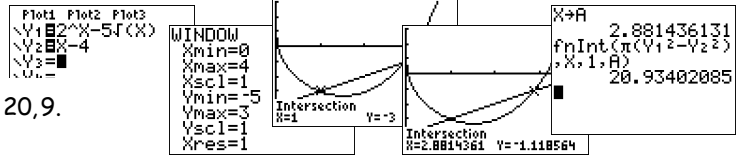
27 $f(x) = g(x)$ (intersect of) $\Rightarrow \sqrt{x} = \frac{1}{2}x \Rightarrow x = \frac{1}{4}x^2 \Rightarrow x^2 = 4x \Rightarrow x = 0 \vee x = 4$ (voldoen).

$$I(L) = \int_0^4 \pi \cdot (f(x))^2 dx - \int_0^4 \pi \cdot (g(x))^2 dx = \int_0^4 \pi \cdot (f(x)^2 - g(x)^2) dx \text{ (fnInt)} \approx 8,4.$$



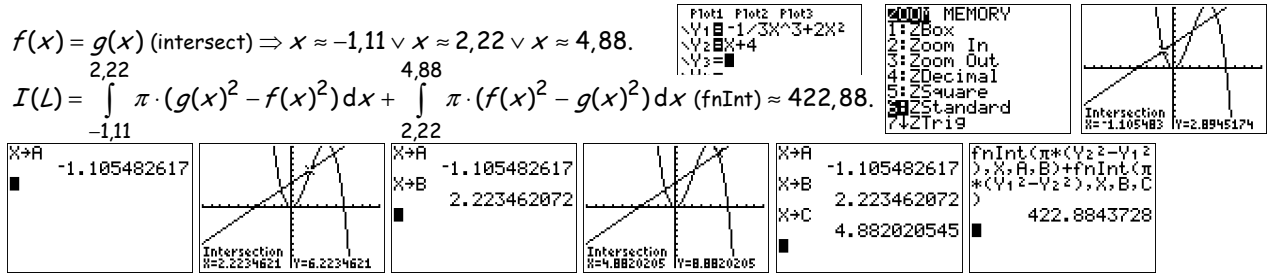
28 $f(x) = g(x)$ (intersect) $\Rightarrow x = 1 \vee x \approx 2,88$.

$$I(L) = \int_1^{2,88} \pi \cdot (f(x)^2 - g(x)^2) dx \text{ (fnInt)} \approx 20,9.$$



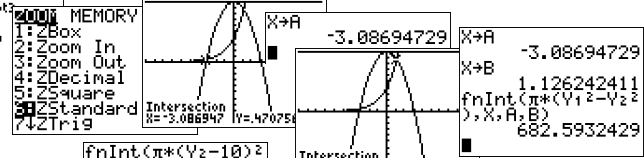
29 $f(x) = g(x)$ (intersect) $\Rightarrow x \approx -1,11 \vee x \approx 2,22 \vee x \approx 4,88$.

$$I(L) = \int_{-1,11}^{2,22} \pi \cdot (g(x)^2 - f(x)^2) dx + \int_{2,22}^{4,88} \pi \cdot (f(x)^2 - g(x)^2) dx \text{ (fnInt)} \approx 422,88.$$



30a $f(x) = g(x)$ (intersect) $\Rightarrow x \approx -3,09 \vee x \approx 1,13$.

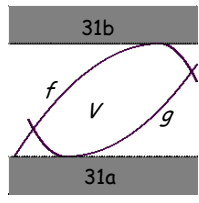
$$I(L) = \int_{-3,09}^{1,13} \pi \cdot (f(x)^2 - g(x)^2) dx \text{ (fnInt)} \approx 682,59.$$



30b $I(M) = \int_{-3,09}^{1,13} \pi \cdot ((g(x)-10)^2 - (f(x)-10)^2) dx \text{ (fnInt)} \approx 569,80.$

31a Er moet gelden $f(x) \geq g(x) \geq 0$ voor x op $[x_A, x_B]$, dus het laagste punt van g ligt op of boven de x -as.

31b Er moet gelden $g(x) \leq f(x) \leq 0$ voor x op $[x_A, x_B]$, dus het hoogste punt van f ligt op of onder de x -as.

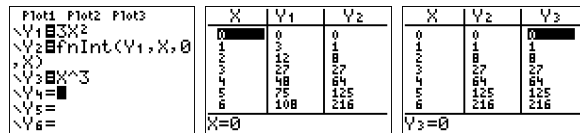


32a $O(p) = O(\text{rechthoek}) + O(\text{driehoek}) = p \cdot b + \frac{1}{2} p \cdot (ap + b - b) = p \cdot b + \frac{1}{2} p \cdot ap = \frac{1}{2} ap^2 + bp.$

32b $O(p) = \frac{1}{2} ap^2 + bp \Rightarrow \frac{dO}{dp} = O'(p) = ap + b.$ Dus $\frac{dO}{dp} = O'(p) = f(p).$

33a

p	1	2	3	4	5
$O(p)$	1	8	27	64	125



33b Voor elke p uit de tabel geldt $O(p) = p^3$. (vermoedelijk is dat voor andere waarden van p ook waar)

33c $O(p) = p^3 \Rightarrow \frac{dO}{dp} = O'(p) = 3p^2.$ Dus $\frac{dO}{dp} = O'(p) = f(p).$

33d $O(p) = \int_0^p f(x) dx = 10 \Rightarrow p^3 = 10 \Rightarrow p = \sqrt[3]{10}.$

34a $F(x) = (x^2 + 1)^6 + 1 \Rightarrow F'(x) = 6 \cdot (x^2 + 1)^5 \cdot 2x = 12x(x^2 + 1)^5.$
Dus $F'(x) = f(x)$ ofwel F is een primitieve van f .

34b $G(x) = (\frac{1}{2}x - \frac{1}{4}) \cdot e^{2x} - 2 \Rightarrow G'(x) = \frac{1}{2} \cdot e^{2x} + (\frac{1}{2}x - \frac{1}{4}) \cdot e^{2x} \cdot 2 = (\frac{1}{2} + x - \frac{1}{2}) \cdot e^{2x} = x \cdot e^{2x}.$
Dus $G'(x) = g(x)$ ofwel G is een primitieve van g .

34c $H(x) = 2\ln(x) + \ln^2(x) + 3 = 2\ln(x) + (\ln(x))^2 + 3 \Rightarrow H'(x) = 2 \cdot \frac{1}{x} + 2\ln(x) \cdot \frac{1}{x} = \frac{2 + 2\ln(x)}{x}.$
Dus $H'(x) = h(x)$ ofwel H is een primitieve van h .

34d $J(x) = \frac{e^{3x} - 10}{2e^x} - 4 \Rightarrow J'(x) = \frac{2e^x \cdot e^{3x} \cdot 3 - (e^{3x} - 10) \cdot 2e^x}{(2e^x)^2} = \frac{3e^{3x} - e^{3x} + 10}{2e^x} = \frac{2e^{3x} + 10}{2e^x} = \frac{e^{3x} + 5}{e^x}.$
Dus $J'(x) = j(x)$ ofwel J is een primitieve van j .

- 35a $F(x) = \frac{1}{5}x^5 \Rightarrow F'(x) = f(x) = \frac{1}{5} \cdot 5x^4 = x^4$. Dus $F(x) = \frac{1}{5}x^5$ is een primitieve van $f(x) = x^4$.
- 35b $G(x) = \frac{1}{4}e^{\overline{4x+1}} \Rightarrow G'(x) = g(x) = \frac{1}{4}e^{4x+1} \cdot 4 = e^{4x+1}$. Dus $G(x) = \frac{1}{4}e^{4x+1}$ is een primitieve van $g(x) = e^{4x+1}$.
- 35c $H(x) = \frac{3^x}{\ln(3)} = \frac{1}{\ln(3)} \cdot 3^x \Rightarrow H'(x) = h(x) = \frac{1}{\ln(3)} \cdot 3^x \cdot \ln(3) = 3^x$. Dus $H(x) = \frac{3^x}{\ln(3)}$ is een primitieve van $h(x) = 3^x$.
- 35d $J(x) = x \cdot \ln(x) \Rightarrow J'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} = \ln(x) + 1$. Dus $J(x) = x \ln(x)$ is een primitieve van $j(x) = \ln(x) + 1$.
- 36a $F(x) = \frac{a}{n+1}x^{n+1} + c$ met $n \neq -1 \Rightarrow F'(x) = f(x) = \frac{a}{n+1} \cdot (n+1) \cdot x^n = ax^n$.
- 36b $F(x) = \frac{g^x}{\ln(g)} + c = \frac{1}{\ln(g)} \cdot g^x + c \Rightarrow F'(x) = f(x) = \frac{1}{\ln(g)} \cdot g^x \cdot \ln(g) = g^x$.
- 36c $F(x) = e^x + c \Rightarrow F'(x) = f(x) = e^x$.
- 36d $F(x) = \ln|x| + c \Rightarrow F'(x) = f(x) = \frac{1}{x}$.
- 36e $F(x) = x \ln(x) - x + c \Rightarrow F'(x) = f(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1 = \ln(x) + 1 - 1 = \ln(x)$.
- 36f $F(x) = \frac{1}{\ln(g)} \cdot (x \ln(x) - x) + c \Rightarrow F'(x) = f(x) = \frac{1}{\ln(g)} \cdot (1 \cdot \ln(x) + x \cdot \frac{1}{x} - 1) = \frac{1}{\ln(g)} \cdot \ln(x) = \frac{\ln(x)}{\ln(g)} = {}^g \log(x)$.
- 37a $f(x) = ax^{-1}$ geeft $F(x) = \frac{a}{-1+1}x^{-1+1} = \frac{a}{0}x^0$. Dit kan niet kloppen omdat $\frac{a}{0}$ niet bestaat en $x^0 = 1$ voor $x \neq 0$.
- 37b $f(x) = x^{-1} = \frac{1}{x} \Rightarrow F(x) = \ln|x| + c$.
- 37c $[a \cdot F(x)]' = a \cdot F'(x) = a \cdot f(x)$.
- 38a $\square f(x) = 6x^2 \Rightarrow F(x) = \frac{6}{3}x^3 + c = 2x^3 + c$.
- 38b $\square f(x) = 2x^3 + 5x^4 \Rightarrow F(x) = \frac{2}{4}x^4 + \frac{5}{5}x^5 + c = \frac{1}{2}x^4 + x^5 + c$.
- 38c $\square f(x) = \frac{x^4 - 2x}{2x^3} = \frac{x^4}{2x^3} - \frac{2x}{2x^3} = \frac{1}{2}x - x^{-2} \Rightarrow F(x) = \frac{1}{2} \cdot \frac{1}{2} \cdot x^2 - \frac{1}{-1}x^{-1} + c = \frac{1}{4}x^2 + \frac{1}{x} + c$.
- 38d $\square f(x) = 10^x \Rightarrow F(x) = \frac{10^x}{\ln(10)} + c$.
- 38e $\square f(x) = 5 \cdot 2^x \Rightarrow F(x) = 5 \cdot \frac{2^x}{\ln(2)} + c = \frac{5 \cdot 2^x}{\ln(2)} + c$.
- 38f $\square f(x) = \frac{x^3 + 2}{x^4} = \frac{x^3}{x^4} + \frac{2}{x^4} = \frac{1}{x} + 2x^{-4} \Rightarrow F(x) = \ln|x| + \frac{2}{-3}x^{-3} + c = \ln|x| - \frac{2}{3x^3} + c$.
- 39a $\square f(x) = x^3 - 3x \Rightarrow F(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2 + c$.
- 39b $\square f(x) = 5e^x \Rightarrow F(x) = 5e^x + c$.
- 39c $\square f(x) = \frac{x^4 - 6}{2x^3} = \frac{x^4}{2x^3} - \frac{6}{2x^3} = \frac{1}{2}x - 3x^{-3} \Rightarrow F(x) = \frac{1}{2} \cdot \frac{1}{2}x^2 - \frac{3}{-2}x^{-2} + c = \frac{1}{4}x^2 + \frac{3}{2x^2} + c$.
- 39d $\square f(x) = 3^x + x^3 \Rightarrow F(x) = \frac{3^x}{\ln(3)} + \frac{1}{4}x^4 + c$.
- 39e $\square f(x) = 2 \ln(x) \Rightarrow F(x) = 2 \cdot (x \ln(x) - x) + c = 2x \ln(x) - 2x + c$.
- 39f $\square f(x) = \ln(2x) = \ln(2) + \ln(x) \Rightarrow F(x) = \ln(2) \cdot x + x \ln(x) - x + c = x \ln(2) + x \ln(x) - x + c$.
- 40a $f(x) = e^{x+1} = e^x \cdot e^1 = e \cdot e^x \Rightarrow F(x) = e \cdot e^x + c = e^{x+1} + c$ of $f(x) = e^{\overline{x+1}} \Rightarrow F(x) = \frac{1}{1} \cdot e^{x+1} + c = e^{x+1} + c$.
- 40b $f(x) = \frac{8}{x^3} = 8x^{-3} \Rightarrow F(x) = \frac{8}{-2}x^{-2} + c = -4x^{-2} + c = -\frac{4}{x^2} + c$.
- 40c $f(x) = \frac{-x^2 + 2x + 3}{x^4} = \frac{-x^2}{x^4} + \frac{2x}{x^4} + \frac{3}{x^4} = -x^{-2} + 2x^{-3} + 3x^{-4} \Rightarrow F(x) = -\frac{1}{-1}x^{-1} + \frac{2}{-2}x^{-2} + \frac{3}{-3}x^{-3} + c = \frac{1}{x} - \frac{1}{x^2} - \frac{1}{x^3} + c$.
- 40d $f(x) = \ln(x\sqrt{x}) = \ln(x^{1\frac{1}{2}}) = 1\frac{1}{2} \cdot \ln(x) \Rightarrow F(x) = 1\frac{1}{2} \cdot (x \ln(x) - x) + c = 1\frac{1}{2}x \ln(x) - 1\frac{1}{2}x + c$.
- 40e $f(x) = {}^2 \log(\frac{1}{x}) = {}^2 \log(x^{-1}) = -2 \log(x) \Rightarrow F(x) = -\frac{1}{\ln(2)} \cdot (x \ln(x) - x) + c = \frac{-x \ln(x) + x}{\ln(2)} + c$.
- 40f $f(x) = 5 \cdot \log(2x) = 5 \cdot \log(2) + 5 \cdot \log(x) \Rightarrow F(x) = 5 \cdot \log(2) \cdot x + 5 \cdot \frac{1}{\ln(10)} \cdot (x \ln(x) - x) + c = 5x \log(2) + \frac{5x \ln(x) - 5x}{\ln(10)} + c$.
- 41a $f(x) = 2x - 3 \Rightarrow F(x) = \frac{2}{2}x^2 - 3x + c = x^2 - 3x + c$.
- 41b $F(x) = x^2 - 3x + c$ door $(1, 2) \Rightarrow 2 = 1^2 - 3 \cdot 1 + c \Rightarrow c = 4$. Dus $F(x) = x^2 - 3x + 4$. $\frac{2-1+3}{4}$

41c $F(x) = x^2 - 3x + c$ (waarvan de grafiek een parabool is) raakt de x -as \Rightarrow
 $F(x) = 0$ (grafiek komt op de x -as) én $F'(x) = f(x) = 0$ (de helling in het punt op de x -as is nul)
 $x^2 - 3x + c = 0$ én $2x - 3 = 0$
 $x^2 - 3x + c = 0$ én $2x = 3$
 $x^2 - 3x + c = 0$ én $x = \frac{3}{2}$
 $(\frac{3}{2})^2 - 3 \cdot \frac{3}{2} + c = 0$ én $x = \frac{3}{2}$
 $c = 3 \cdot \frac{3}{2} - (\frac{3}{2})^2 = \frac{9}{2} - \frac{9}{4} = \frac{18}{4} - \frac{9}{4} = \frac{9}{4} \Rightarrow F(x) = x^2 - 3x + \frac{9}{4}$.

42 $f(x) = (x^2 - 1)^2 = x^4 - 2x^2 + 1 \Rightarrow F(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + c$.
 $F(x)$ door $(1, 7) \Rightarrow 7 = \frac{1}{5} \cdot 1^5 - \frac{2}{3} \cdot 1^3 + 1 + c \Rightarrow 7 - \frac{1}{5} + \frac{2}{3} - 1 = c \Rightarrow c = 6\frac{7}{15}$. Dus $F(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + 6\frac{7}{15}$.

43a $f(x) = x^2 \Rightarrow F(x) = \frac{1}{3}x^3 + c$.

43b $O(x) = F(x) = \frac{1}{3}x^3 + c$ én $O(0) = 0 \Rightarrow \frac{1}{3} \cdot 0^3 + c = 0 \Rightarrow c = 0$.

43c $O(x) = \frac{1}{3}x^3$ én $O(p) = 10 \Rightarrow \frac{1}{3} \cdot p^3 = 10 \Rightarrow p^3 = 30 \Rightarrow p = \sqrt[3]{30}$.

44a $f(x) = 0 \Rightarrow 3x^2 - x^3 = 0 \Rightarrow x^2 \cdot (3 - x) = 0 \Rightarrow x = 0 \vee x = 3$.
 $O(V) = \int_0^3 (3x^2 - x^3) dx = [x^3 - \frac{1}{4}x^4]_0^3 = 3^3 - \frac{1}{4} \cdot 3^4 - (0^3 - \frac{1}{4} \cdot 0^4) = 6\frac{3}{4}$.

44b $\int_0^p (3x^2 - x^3) dx = \frac{1}{2} \cdot 6\frac{3}{4}$
 $[x^3 - \frac{1}{4}x^4]_0^p = 3\frac{3}{8}$
 $p^3 - \frac{1}{4} \cdot p^4 - 0 = 3\frac{3}{8}$ (intersect) $\Rightarrow p \approx 1,84$.

44c $I(L) = \int_0^3 \pi(3x^2 - x^3)^2 dx = \int_0^3 \pi(9x^4 - 6x^5 + x^6) dx = [\pi(\frac{9}{5}x^5 - x^6 + \frac{1}{7}x^7)]_0^3 = \pi(\frac{9}{5} \cdot 3^5 - 3^6 + \frac{1}{7} \cdot 3^7) - 0 = \frac{729}{35} \pi$.

44d $\int_0^q \pi \cdot f(x)^2 dx = [\pi(\frac{9}{5}x^5 - x^6 + \frac{1}{7}x^7)]_0^q = \frac{1}{2} \cdot \frac{729}{35} \pi$
 $\pi(\frac{9}{5} \cdot q^5 - q^6 + \frac{1}{7} \cdot q^7) - 0 = \frac{1}{2} \cdot \frac{729}{35} \pi$ (intersect) $\Rightarrow q \approx 1,91$.

45a $x^2 = 6 - x$ (intersect of)
 $x^2 + x - 6 = 0$
 $(x + 3)(x - 2) = 0$
 $x = -3 \vee x = 2$.

$O(V) = \int_{-3}^2 (g(x) - f(x)) dx = \int_{-3}^2 (6 - x - x^2) dx$ (niet met fnInt)
 $= [6x - \frac{1}{2}x^2 - \frac{1}{3}x^3]_{-3}^2 = 6 \cdot 2 - \frac{1}{2} \cdot 2^2 - \frac{1}{3} \cdot 2^3 - (6 \cdot (-3) - \frac{1}{2} \cdot (-3)^2 - \frac{1}{3} \cdot (-3)^3) = 20\frac{5}{6}$

45b $\int_{-3}^p (6 - x - x^2) dx = \frac{1}{2} \cdot 20\frac{5}{6}$
 $[6x - \frac{1}{2}x^2 - \frac{1}{3}x^3]_{-3}^p = 6 \cdot p - \frac{1}{2} \cdot p^2 - \frac{1}{3} \cdot p^3 - (6 \cdot (-3) - \frac{1}{2} \cdot (-3)^2 - \frac{1}{3} \cdot (-3)^3) = \frac{1}{2} \cdot 20\frac{5}{6}$ (intersect) $\Rightarrow p = -0,5$.

45c $I(L) = \int_{-3}^2 \pi \cdot (6 - x)^2 dx - \int_{-3}^2 \pi \cdot (x^2)^2 dx = \int_{-3}^2 \pi \cdot (36 - 12x + x^2) dx - \int_{-3}^2 \pi \cdot x^4 dx$
 $= [\pi \cdot (36x - 6x^2 + \frac{1}{3}x^3)]_{-3}^2 - [\pi \cdot \frac{1}{5}x^5]_{-3}^2$
 $= \pi \cdot (36 \cdot 2 - 6 \cdot 2^2 + \frac{1}{3} \cdot 2^3) - \pi \cdot (36 \cdot (-3) - 6 \cdot (-3)^2 + \frac{1}{3} \cdot (-3)^3) - (\pi \cdot \frac{1}{5} \cdot 2^5 - \pi \cdot \frac{1}{5} \cdot (-3)^5) = \frac{500}{3} \pi$.

46a $f(x) = -\frac{1}{2} \Rightarrow \frac{x^2+x+1}{x} = -\frac{1}{2} \Rightarrow x^2+x+1 = -\frac{1}{2}x \Rightarrow x^2 + 2\frac{1}{2}x + 1 = 0 \Rightarrow (x+2)(x+\frac{1}{2}) = 0 \Rightarrow x = -2 \vee x = -\frac{1}{2}$.

$$O(V) = \int_{-2}^{-\frac{1}{2}} \left(\frac{x^2+x+1}{x} - (-\frac{1}{2}) \right) dx = \int_{-2}^{-\frac{1}{2}} \left(x+1+\frac{1}{x}+1\frac{1}{2} \right) dx = \int_{-2}^{-\frac{1}{2}} \left(x+\frac{1}{x}+2\frac{1}{2} \right) dx = \left[\frac{1}{2}x^2 + \ln|x| + 2\frac{1}{2}x \right]_{-2}^{-\frac{1}{2}}$$

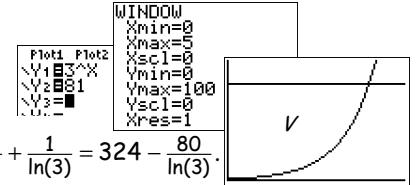
$$= \frac{1}{2} \cdot \left(-\frac{1}{2}\right)^2 + \ln\left(\frac{1}{2}\right) + 2\frac{1}{2} \cdot \left(-\frac{1}{2}\right) - \left(\frac{1}{2} \cdot (-2)^2 + \ln(2) + 2\frac{1}{2} \cdot (-2) \right) = \frac{1}{8} + \ln\left(\frac{1}{2}\right) - 1\frac{1}{4} - (2 + \ln(2) - 5) = 1\frac{7}{8} + \ln\left(\frac{1}{2}\right) - \ln(2).$$

46b $O(W) = \int_1^p \left(\frac{x^2+x+1}{x} - (x+1) \right) dx = \int_1^p \left(x+1+\frac{1}{x}-x-1 \right) dx = \int_1^p \frac{1}{x} dx = [\ln|x|]_1^p = \ln(p) - \ln(1) = \ln(p)$.

$O(W) = 2 \Rightarrow \ln(p) = 2 \Rightarrow p = e^2$.

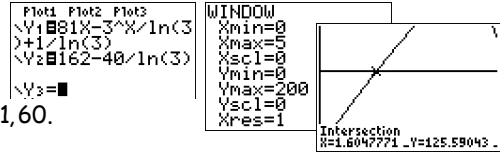
47a $f(x) = 81 \Rightarrow 3^x = 81 = 3^4 \Rightarrow x = 4$.

$$O(V) = \int_0^4 (81 - 3^x) dx = \left[81x - \frac{3^x}{\ln(3)} \right]_0^4 = 81 \cdot 4 - \frac{3^4}{\ln(3)} - \left(81 \cdot 0 - \frac{3^0}{\ln(3)} \right) = 324 - \frac{81}{\ln(3)} + \frac{1}{\ln(3)} = 324 - \frac{80}{\ln(3)}$$



47b $\int_0^a (81 - 3^x) dx = \frac{1}{2} \cdot \left(324 - \frac{80}{\ln(3)} \right)$

$$\left[81x - \frac{3^x}{\ln(3)} \right]_0^a = 81a - \frac{3^a}{\ln(3)} + \frac{1}{\ln(3)} = 162 - \frac{40}{\ln(3)} \text{ (intersect)} \Rightarrow a \approx 1,60$$



48 $f(x) = 8 \Rightarrow \frac{8}{x^2} = \frac{8}{1} \Rightarrow 8x^2 = 8 \Rightarrow x^2 = 1 \Rightarrow x = -1 \text{ (zoeken we niet)} \vee x = 1$.

$$O(V) = 8 \cdot 1 + \int_1^8 \frac{8}{x^2} dx = 8 \cdot 1 + \int_1^8 8x^{-2} dx = 8 + \left[-8x^{-1} \right]_1^8 = 8 + \left[-\frac{8}{x} \right]_1^8 = 8 + \left(-\frac{8}{8} - \left(-\frac{8}{1} \right) \right) = 8 + (-1 + 8) = 8 + 7 = 15$$

$$O(V_1) = \frac{2}{3} \cdot 15 = 10 \Rightarrow 8 + \int_1^a \frac{8}{x^2} dx = 8 + \left[-\frac{8}{x} \right]_1^a = 8 + \left(-\frac{8}{a} - \left(-\frac{8}{1} \right) \right) = 8 - \frac{8}{a} + 8 = 16 - \frac{8}{a} = 10 \Rightarrow 6 = \frac{8}{a} \Rightarrow 6a = 8 \Rightarrow a = \frac{8}{6} = \frac{4}{3}$$

49 $I(L_1 + L_2) = \int_2^8 \pi(\sqrt{x-2})^2 dx = \int_2^8 \pi(x-2) dx = \left[\pi\left(\frac{1}{2}x^2 - 2x\right) \right]_2^8 = \pi(32-16) - \pi(2-4) = 16\pi - (-2\pi) = 18\pi$.

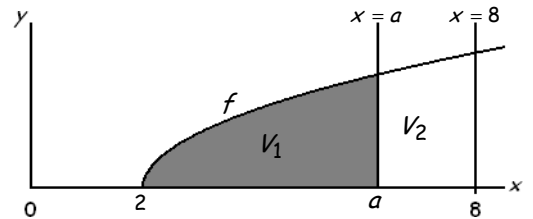
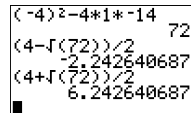
$$I(L_1) = \int_2^a \pi(\sqrt{x-2})^2 dx = \frac{1}{2} \cdot 18\pi \text{ geeft } \left[\pi\left(\frac{1}{2}x^2 - 2x\right) \right]_2^a = 9\pi$$

$$\pi\left(\frac{1}{2}a^2 - 2a\right) - \pi(2-4) = 9\pi \Rightarrow \frac{1}{2}a^2 - 2a + 2 = 9$$

$$a^2 - 4a + 4 = 18 \Rightarrow a^2 - 4a - 14 = 0$$

$$D = 16 - 4 \cdot 1 \cdot (-14) = 72$$

$$a = \frac{4 - \sqrt{72}}{2} \text{ (vold. niet)} \vee a = \frac{4 + \sqrt{72}}{2} \text{ (voldoet)} = \frac{4 + 6\sqrt{2}}{2} = 2 + 3\sqrt{2}$$



50a $F(x) = a(3x+1)^6 \Rightarrow F'(x) = f(x) = 6a(3x+1)^5 \cdot 3 = 18a(3x+1)^5$.

50b $(3x+1)^5 = 18a(3x+1)^5 \Rightarrow 1 = 18a \Rightarrow \frac{1}{18} = a$.

51 $G(x) = \frac{1}{a} F(ax+b) \Rightarrow G'(x) = g(x) = \frac{1}{a} F'(ax+b) \cdot a = F'(ax+b) = f(ax+b)$.

52a $f(x) = (2x-1)^6 \Rightarrow F(x) = \frac{1}{2} \cdot \frac{1}{7} \cdot (2x-1)^7 + c = \frac{1}{14}(2x-1)^7 + c$.

52b $g(x) = \frac{1}{(3x+4)^3} = (3x+4)^{-3} \Rightarrow G(x) = \frac{1}{3} \cdot \frac{1}{-2} \cdot (3x+4)^{-2} + c = -\frac{1}{6} \cdot \frac{1}{(3x+4)^2} + c = \frac{-1}{6(3x+4)^2} + c$.

52c $h(x) = 4\sqrt{3-2x} = 4 \cdot (3-2x)^{\frac{1}{2}} \Rightarrow H(x) = 4 \cdot \frac{1}{-2} \cdot \frac{1}{\frac{1}{2}} \cdot (3-2x)^{\frac{1}{2}} + c = -\frac{4}{3} \cdot (3-2x) \cdot \sqrt{3-2x} + c$.

52d $j(x) = \frac{2}{\sqrt{1-x}} = 2 \cdot (1-x)^{-\frac{1}{2}} \Rightarrow J(x) = 2 \cdot \frac{1}{-1} \cdot \frac{1}{\frac{1}{2}} \cdot (1-x)^{\frac{1}{2}} + c = -4 \cdot (1-x)^{\frac{1}{2}} + c = -4 \cdot \sqrt{1-x} + c$.

53a $f(x) = \frac{1}{x-1} \Rightarrow F(x) = \frac{1}{1} \cdot \ln|x-1| + c = \ln|x-1| + c$.

53b $f(x) = \frac{3}{2x-5} = 3 \cdot \frac{1}{2x-5} \Rightarrow F(x) = 3 \cdot \frac{1}{2} \cdot \ln|2x-5| + c = \frac{3}{2} \cdot \ln|2x-5| + c$.

53c $f(x) = e^{\sqrt{4x-1}} \Rightarrow F(x) = \frac{1}{4} \cdot e^{4x-1} + c.$

53d $f(x) = \ln(\sqrt{4x-1}) \Rightarrow F(x) = \frac{1}{4} \cdot ((4x-1) \cdot \ln(4x-1) - (4x-1)) + c = (x - \frac{1}{4}) \cdot \ln(4x-1) - x + \frac{1}{4} + c.$

53e $f(x) = (2x+1) \cdot \sqrt{2x+1} = ((2x+1))^{\frac{3}{2}} \Rightarrow F(x) = \frac{1}{2} \cdot \frac{1}{\frac{3}{2}} (2x+1)^{\frac{5}{2}} + c = \frac{1}{5} \cdot (2x+1)^2 \cdot \sqrt{2x+1} + c.$

53f $f(x) = 2^{\sqrt{3x}} \Rightarrow F(x) = \frac{1}{3} \cdot \frac{1}{\ln(2)} \cdot 2^{3x} + c = \frac{2^{3x}}{3\ln(2)} + c.$

53g $f(x) = 3^{\sqrt{2-5x}} \Rightarrow F(x) = \frac{1}{-5} \cdot \frac{1}{\ln(3)} \cdot 3^{2-5x} + c = -\frac{3^{2-5x}}{5\ln(3)} + c.$

53h $f(x) = 2 \log(5x+3) = \frac{\ln(5x+3)}{\ln(2)} \Rightarrow F(x) = \frac{1}{5} \cdot \frac{1}{\ln(2)} \cdot ((5x+3) \cdot \ln(5x+3) - (5x+3)) + c.$

54a $f(x) = g(x) \Rightarrow \frac{5}{2x+1} = \frac{5}{10-4x} \Rightarrow 5 \cdot (2x+1) = 5 \cdot (10-4x) \Rightarrow 2x+1 = 10-4x \Rightarrow 6x = 9 \Rightarrow x = \frac{9}{6} = 1\frac{1}{2}.$

$$O(V) = \int_0^{\frac{1}{2}} \frac{5}{10-4x} dx + \int_{\frac{1}{2}}^2 \frac{5}{2x+1} dx = \left[5 \cdot \frac{1}{-4} \cdot \ln|10-4x| \right]_0^{\frac{1}{2}} + \left[5 \cdot \frac{1}{2} \cdot \ln|2x+1| \right]_{\frac{1}{2}}^2$$

$$= -\frac{5}{4} \cdot \ln(4) + \frac{5}{4} \cdot \ln(10) + \frac{5}{2} \cdot \ln(5) - \frac{5}{2} \cdot \ln(4) = \frac{5}{4} \cdot \ln(10) + \frac{5}{2} \cdot \ln(5) - \frac{15}{4} \cdot \ln(4).$$

54b $I(L) = \int_0^{\frac{1}{2}} \pi \cdot \left(\frac{5}{10-4x}\right)^2 dx + \int_{\frac{1}{2}}^2 \pi \cdot \left(\frac{5}{2x+1}\right)^2 dx = \int_0^{\frac{1}{2}} 25\pi \cdot (10-4x)^{-2} dx + \int_{\frac{1}{2}}^2 25\pi \cdot (2x+1)^{-2} dx$

$$= \left[25\pi \cdot \frac{1}{-4} \cdot \frac{1}{-1} \cdot (10-4x)^{-1} \right]_0^{\frac{1}{2}} + \left[25\pi \cdot \frac{1}{2} \cdot \frac{1}{-1} \cdot (2x+1)^{-1} \right]_{\frac{1}{2}}^2 = \left[\frac{25\pi}{4 \cdot (10-4x)} \right]_0^{\frac{1}{2}} + \left[-\frac{25\pi}{2 \cdot (2x+1)} \right]_{\frac{1}{2}}^2$$

$$= \frac{25\pi}{4 \cdot 4} - \frac{25\pi}{4 \cdot 10} - \frac{25\pi}{2 \cdot 5} + \frac{25\pi}{2 \cdot 4} = \frac{25}{16} \pi \cdot \frac{25 \cdot 16 - 25 \cdot 40 - 25 \cdot 16 + 25 \cdot 8}{25 \cdot 16}$$

55a De regel gaat over functies van de vorm $f(ax+b)$ en g is van de vorm $f(ax^2+b)$.

55b $G(x) = a(\sqrt{4x^2-1})^{\frac{1}{2}} \Rightarrow G'(x) = a \cdot \frac{1}{2} \cdot (4x^2-1)^{-\frac{1}{2}} \cdot 8x = 4ax \cdot \sqrt{4x^2-1}.$

Er is geen waarde van a waarvoor $G'(x) = \sqrt{4x^2-1}.$

55c $H(x) = a \cdot (4x^2-1)^{\frac{1}{2}} + c \Rightarrow h(x) = 4ax \cdot \sqrt{4x^2-1}$ (zie 55b). Dus $4ax = x \Rightarrow a = \frac{1}{4}.$

Dus $h(x) = x \cdot \sqrt{4x^2-1}$ heeft als primitieven $H(x) = \frac{1}{12} \cdot (4x^2-1)^{\frac{3}{2}} + c = \frac{1}{12} \cdot (4x^2-1) \cdot \sqrt{4x^2-1} + c.$

56a $I(L) = \int_0^6 \pi \cdot \left(\frac{1}{2}x\right)^2 dx = \int_0^6 \pi \cdot \frac{1}{4}x^2 dx = \left[\pi \cdot \frac{1}{4} \cdot \frac{1}{3} \cdot x^3 \right]_0^6 = \frac{1}{12} \pi \cdot 6^3 - \frac{1}{12} \pi \cdot 0 = 18\pi.$

56b L is een kegel. De straal van de grondcirkel is 3 en de hoogte is 6.

57 $I(\text{kegel}) = \frac{1}{3} Gh = \frac{1}{3} \pi r^2 h = \frac{1}{3} \cdot \pi \cdot (3 \cdot 6)^2 \cdot 6 = \frac{1}{3} \cdot \pi \cdot 36 \cdot 6 = 32\pi.$

$$I(\text{kegeltop}) = \frac{1}{3} \cdot \pi \cdot \left(\frac{2}{3}p\right)^2 \cdot p = \frac{1}{3} \cdot \pi \cdot \frac{4}{9}p^2 \cdot p = \frac{4}{27} \pi p^3.$$

$$K_p = 24\pi \Rightarrow 32\pi - \frac{4}{27} \pi p^3 = 24\pi \Rightarrow -\frac{4}{27} \pi p^3 = -8\pi \Rightarrow \frac{4}{27} p^3 = 8 \Rightarrow p^3 = 8 \cdot \frac{27}{4} = 54 \Rightarrow p = \sqrt[3]{54} = \sqrt[3]{2 \cdot 3 \cdot 3 \cdot 3} = 3 \cdot \sqrt[3]{2}.$$

58 $I(\text{kegel}) = \frac{1}{3} Gh = \frac{1}{3} \cdot \pi \cdot \left(\frac{3}{2} \cdot 8\right)^2 \cdot 8 = \frac{1}{3} \cdot \pi \cdot 36 \cdot 8 = 384\pi.$

$$I(\text{kegeltop}) = \frac{1}{3} \cdot \pi \cdot \left(\frac{3}{2}p\right)^2 \cdot p = \frac{1}{3} \cdot \pi \cdot \frac{9}{4}p^2 \cdot p = \frac{3}{4} \pi p^3.$$

$$K_p = 378\pi \Rightarrow 384\pi - \frac{3}{4} \pi p^3 = 378\pi \Rightarrow -\frac{3}{4} \pi p^3 = -6\pi \Rightarrow \frac{3}{4} p^3 = 6 \Rightarrow p^3 = 6 \cdot \frac{4}{3} = 8 \Rightarrow p = \sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2} = 2.$$

59 De cirkel met straal r en middelpunt $(0, 0)$ heeft vergelijking $x^2 + y^2 = r^2$ ofwel $y^2 = r^2 - x^2.$

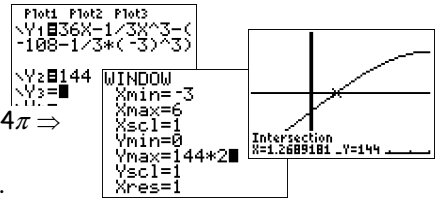
$$I(\text{bol}) = \int_{-r}^r \pi y^2 dx = 2 \cdot \int_0^r \pi y^2 dx = 2 \cdot \int_0^r \pi (r^2 - x^2) dx = 2 \cdot \left[\pi (r^2 x - \frac{1}{3} x^3) \right]_0^r = 2 \cdot (\pi (r^3 - \frac{1}{3} r^3)) = 2\pi \cdot \frac{2}{3} r^3 = \frac{4}{3} \pi r^3.$$

60 $I = \int_{\frac{1}{3}r}^r \pi y^2 dx = \int_{\frac{1}{3}r}^r \pi (r^2 - x^2) dx = \left[\pi (r^2 x - \frac{1}{3} x^3) \right]_{\frac{1}{3}r}^r = \pi (r^3 - \frac{1}{3} r^3) - \pi (\frac{1}{3} r^3 - \frac{1}{3} \cdot (\frac{1}{3} r)^3) = \pi \cdot \frac{2}{3} r^3 - \pi \cdot \frac{26}{81} r^3 = \frac{28}{81} \pi r^3.$

61 $I(\text{bol}) = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 6^3 = 288\pi$. Nu is $I(S_p) = \frac{1}{2} \cdot 288\pi = 144\pi \Rightarrow$

$$\int_{-3}^p \pi y^2 dx = 144\pi \Rightarrow \int_{-3}^p \pi(36 - x^2) dx = 144\pi \Rightarrow \left[\pi(36x - \frac{1}{3}x^3) \right]_{-3}^p = 144\pi \Rightarrow$$

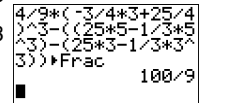
$$\pi(36p - \frac{1}{3}p^3) - \pi(-108 - \frac{1}{3} \cdot -27) = 144\pi. \text{ Intersect geeft dan } p \approx 1,27.$$



62 k snijden met de x -as ($y=0$) $\Rightarrow -\frac{3}{4}x + 6\frac{1}{4} = 0 \Rightarrow -\frac{3}{4}x = -\frac{25}{4} \Rightarrow x = -\frac{25}{4} \cdot -\frac{4}{3} = \frac{25}{3}$.

$$I(L) = \int_3^{\frac{25}{3}} \pi(-\frac{3}{4}x + 6\frac{1}{4})^2 dx - \int_3^{\frac{25}{3}} \pi(25 - x^2) dx = \left[\pi \cdot -\frac{4}{3} \cdot \frac{1}{3} \cdot (-\frac{3}{4}x + 6\frac{1}{4})^3 \right]_3^{\frac{25}{3}} - \left[\pi(25x - \frac{1}{3}x^3) \right]_3^{\frac{25}{3}}$$

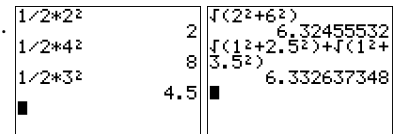
$$= -\frac{4}{9}\pi \cdot 0^3 - -\frac{4}{9}\pi \cdot (-\frac{3}{4} \cdot 3 + 6\frac{1}{4})^3 - (\pi \cdot (25 \cdot \frac{25}{3} - \frac{1}{3} \cdot 5^3) - \pi \cdot (25 \cdot 3 - \frac{1}{3} \cdot 3^3)) = \frac{100}{9}\pi = 11\frac{1}{9}\pi.$$



63a $A(2, 2)$ en $B(4, 8) \Rightarrow AB = \sqrt{(4-2)^2 + (8-2)^2} = \sqrt{2^2 + 6^2} = \sqrt{4+36} = \sqrt{40} \approx 6,32$.

63b $C(3; 4, 5) \Rightarrow AC + CB = \sqrt{1^2 + 2,5^2} + \sqrt{1^2 + 3,5^2} \approx 6,33$.

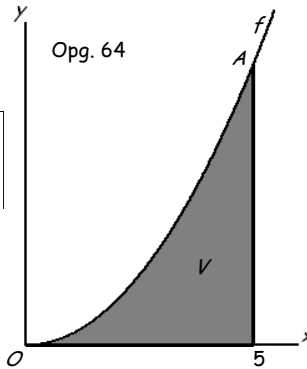
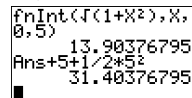
Deze tweede benadering is beter omdat $AB < AC + CB < \text{boog } AB$.



64 $f(x) = \frac{1}{2}x^2 \Rightarrow f'(x) = x$.

boog $OA = \int_0^5 \sqrt{1+x^2} dx$ (fnInt) $\approx 13,904$.

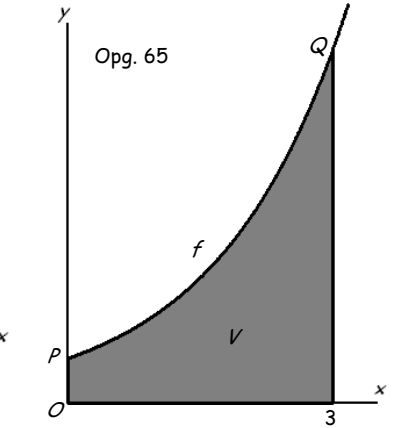
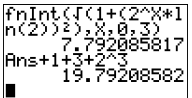
omtrek $\approx 13,904 + 5 + \frac{1}{2} \cdot 5^2 \approx 31,40$.



65 $f(x) = 2^x \Rightarrow f'(x) = 2^x \cdot \ln(2)$.

boog $PQ = \int_0^3 \sqrt{1+(2^x \cdot \ln(2))^2} dx$ (fnInt) $\approx 7,792$.

omtrek $\approx 7,792 + 1 + 3 + 2^3 \approx 19,79$.

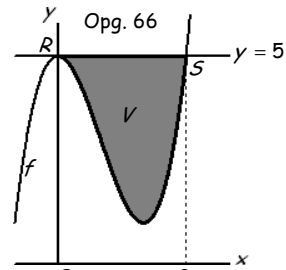
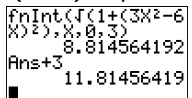


66 $x^3 - 3x^2 + 5 = 5 \Rightarrow x^2(x-3) = 0 \Rightarrow x=0 \vee x=3$.

$f(x) = x^3 - 3x^2 + 5 \Rightarrow f'(x) = 3x^2 - 6x$.

boog $RS = \int_0^3 \sqrt{1+(3x^2-6x)^2} dx$ (fnInt) $\approx 8,81$.

omtrek $\approx 8,81 + 3 \approx 11,81$.



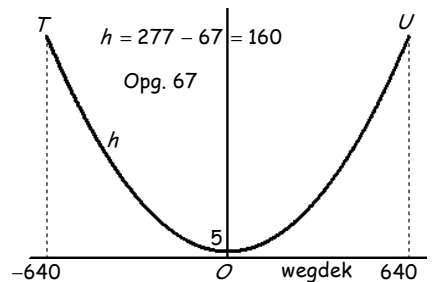
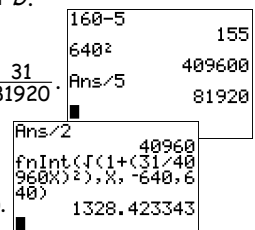
67 De formule van de kabel is van de vorm $h(x) = ax^2 + b$.

$(0, 5)$ op de parabool $\Rightarrow b = 5$.

$h(x) = ax^2 + 5 \Rightarrow 160 = a \cdot 640^2 + 5 \Rightarrow a = \frac{155}{640^2} = \frac{31}{81920}$

$h(x) = \frac{31}{81920}x^2 + 5 \Rightarrow h'(x) = \frac{31}{40960}x$.

boog $TU = \int_{-640}^{640} \sqrt{1 + (\frac{31}{40960}x)^2} dx$ (fnInt) ≈ 1328 (m).

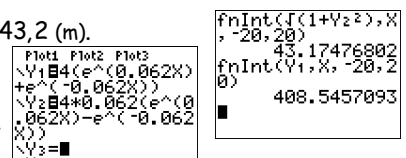
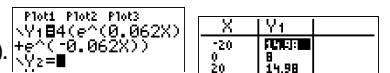


68a $y(x) = y(-x) \Rightarrow$ hoogte van bevestiging aan de palen is $y(20) = y(-20) \approx 15,0$ (m).

Niet gevraagd wordt: de laagste hoogte van de kabel boven het wegdek $y(0) = 4 \cdot (e^0 + e^0) = 4 \cdot (1+1) = 4 \cdot 2 = 8$ (m).

68b $y = 4(e^{0,062x} + e^{-0,062x}) \Rightarrow y' = 4(e^{0,062x} \cdot 0,062 + e^{-0,062x} \cdot -0,062) = 0,248(e^{0,062x} - e^{-0,062x})$.

Lengte van één kabel: $\int_{-20}^{20} \sqrt{1 + (0,248(e^{0,062x} - e^{-0,062x}))^2} dx$ (fnInt) $\approx 43,2$ (m).



68c Oppervlakte van één net: $\int_{-20}^{20} 4(e^{0,062x} + e^{-0,062x}) dx$ (fnInt) ≈ 409 (m²).

- 69 Voor de straal van de onderste cilinder moet je de grafiek van f snijden met de lijn $y = \frac{1}{2}$.
 $f(x) = \frac{1}{2} \Rightarrow \sqrt{x} = \frac{1}{2}$ (kwadrateren) $\Rightarrow x = \frac{1}{4}$. Dus de straal van de onderste cilinder is $\frac{1}{4}$.
 Voor de straal van de bovenste cilinder moet je de grafiek van f snijden met de lijn $y = 1\frac{1}{2}$.
 $f(x) = 1\frac{1}{2} \Rightarrow \sqrt{x} = 1\frac{1}{2}$ (kwadrateren) $\Rightarrow x = 2\frac{1}{4}$. Dus de straal van de bovenste cilinder is $2\frac{1}{4}$.

0.5^2	.25
1.5^2	2.25

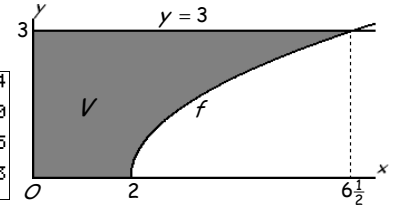
70a $y = \sqrt{2x-4} \Rightarrow y^2 = 2x-4 \Rightarrow 2x = y^2+4 \Rightarrow x = \frac{1}{2}y^2+2 \Rightarrow x^2 = (\frac{1}{2}y^2+2)^2 = \frac{1}{4}y^4+2y^2+4$.

$$I(L) = \int_0^3 \pi x^2 dy = \int_0^3 \pi (\frac{1}{4}y^4 + 2y^2 + 4) dy$$

$$= \left[\pi \cdot (\frac{1}{4} \cdot \frac{1}{5} y^5 + 2 \cdot \frac{1}{3} y^3 + 4y) \right]_0^3 = \left[\pi \cdot (\frac{1}{20} y^5 + \frac{2}{3} y^3 + 4y) \right]_0^3$$

$$= \pi \cdot (\frac{1}{20} \cdot 3^5 + \frac{2}{3} \cdot 3^3 + 4 \cdot 3) - \pi \cdot 0 = \frac{843}{20} \pi = 42 \frac{3}{20} \pi.$$

$3^5/20+2/3*3^3+4$	$843/20$
$*3\pi$	$843/20$
	42.15
	42.15
	3



70b $y = 3 \Rightarrow \sqrt{2x-4} = 3 \Rightarrow 2x-4 = 9 \Rightarrow 2x = 13 \Rightarrow x = 6\frac{1}{2}$; $y = 0 \Rightarrow \sqrt{2x-4} = 0 \Rightarrow 2x-4 = 0 \Rightarrow 2x = 4 \Rightarrow x = 2$.

$$I(M) = I(\text{cilinder}) - \int_2^{6\frac{1}{2}} \pi y^2 dx = \pi \cdot 3^2 \cdot 6\frac{1}{2} - \int_2^{6\frac{1}{2}} \pi (2x-4) dx = 58\frac{1}{2} \pi - \left[\pi \cdot (x^2 - 4x) \right]_2^{6\frac{1}{2}}$$

$$= 58\frac{1}{2} \pi - \left(\pi \cdot (6\frac{1}{2}^2 - 4 \cdot 6\frac{1}{2}) - \pi \cdot (2^2 - 4 \cdot 2) \right) = 38\frac{1}{4} \pi.$$

$9*6.5$	58.5
$\pi * (6.5^2 - 4*6.5)$	38.25
$-(2^2 - 4*2)$	

71a $I(L) = \int_0^4 \pi x^2 dy = \int_0^4 \pi y dy = \left[\pi \cdot \frac{1}{2} y^2 \right]_0^4 = \pi \cdot \frac{1}{2} \cdot 4^2 - \pi \cdot 0 = 8\pi.$

71b $I(L_q) = \int_0^q \pi y dy = \left[\pi \cdot \frac{1}{2} y^2 \right]_0^q = \frac{1}{2} q^2 \pi$ en $I(L_p) = \int_0^p \pi y^2 dx = \int_0^p \pi x^4 dx = \left[\pi \cdot \frac{1}{5} x^5 \right]_0^p = \pi \cdot \frac{1}{5} \cdot p^5 - \pi \cdot 0 = \frac{1}{5} p^5 \pi.$

$P(p, q)$ ligt op de grafiek van de parabool $y = x^2 \Rightarrow q = p^2$.

$$I(L_q) = I(L_p) \text{ met } q = p^2 \Rightarrow \frac{1}{2} q^2 \pi = \frac{1}{5} p^5 \pi \text{ met } q = p^2 \Rightarrow \frac{1}{2} p^4 \pi = \frac{1}{5} p^5 \pi \Rightarrow \frac{1}{2} p^4 = \frac{1}{5} p^5 \Rightarrow p = 0 \vee \frac{1}{2} = \frac{1}{5} p.$$

$p = 0$ geeft geen omwentelingslichaam (dus voldoet niet) en $\frac{1}{2} = \frac{1}{5} p$ geeft $p = \frac{5}{2}$ en $q = p^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$.

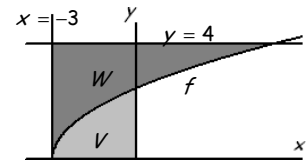
72a $I(L) = \int_{-3}^0 \pi y^2 dx = \int_{-3}^0 \pi \cdot (\sqrt{2x+6})^2 dx = \int_{-3}^0 \pi (2x+6) dx = \left[\pi (x^2 + 6x) \right]_{-3}^0 = \pi \cdot 0 - \pi \cdot (9 - 18) = -9\pi = 9\pi.$

72b $f(0) = \sqrt{6}$.

$$y = \sqrt{2x+6} \Rightarrow y^2 = 2x+6 \Rightarrow 2x = y^2 - 6 \Rightarrow x = \frac{1}{2}y^2 - 3 \Rightarrow x^2 = \frac{1}{4}y^4 - 3y^2 + 9.$$

$$I(M) = \int_0^{\sqrt{6}} \pi x^2 dy = \int_0^{\sqrt{6}} \pi (\frac{1}{4}y^4 - 3y^2 + 9) dy = \left[\pi (\frac{1}{20}y^5 - y^3 + 9y) \right]_0^{\sqrt{6}}$$

$$= \pi \cdot (\frac{1}{20} \cdot \sqrt{6}^5 - \sqrt{6}^3 + 9 \cdot \sqrt{6}) - \pi \cdot 0 = \pi \cdot (\frac{1}{20} \cdot 6^2 \cdot \sqrt{6} - 6 \cdot \sqrt{6} + 9 \cdot \sqrt{6}) = \frac{24}{5} \pi \cdot \sqrt{6} = 4\frac{4}{5} \pi \cdot \sqrt{6}.$$



$36/20-6+9$	4.8
$\text{Ans} \cdot \pi$	$24/5$

72c $y = \sqrt{2x+6} \xrightarrow{\text{translatie (3,0)}} y = \sqrt{2(x-3)+6} = \sqrt{2x-6+6} = \sqrt{2x}.$

$$y = \sqrt{2x} \Rightarrow y^2 = 2x \Rightarrow x = \frac{1}{2}y^2 \Rightarrow x^2 = \frac{1}{4}y^4.$$

$$I(N) = \int_0^4 \pi x^2 dy = \int_0^4 \pi \cdot \frac{1}{4} y^4 dy = \left[\pi \cdot \frac{1}{20} y^5 \right]_0^4 = \pi \cdot \frac{1}{20} \cdot 4^5 - \pi \cdot 0 = \frac{256}{5} \pi = 51\frac{1}{5} \pi.$$

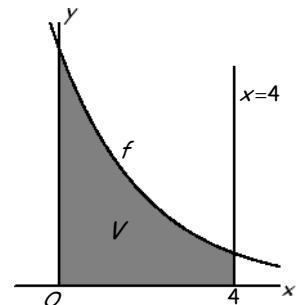
$4^5/20 \cdot \pi$	$256/5$
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73a $O(V) = \int_0^4 2e^{-\frac{1}{2}x+1} dx = \left[2 \cdot \frac{1}{-\frac{1}{2}} \cdot e^{-\frac{1}{2}x+1} \right]_0^4 = \left[-4 \cdot e^{-\frac{1}{2}x+1} \right]_0^4 = -4 \cdot e^{-2} - (-4 \cdot e^1) = 4e - \frac{4}{e}.$

73b $f(x) = 2e^{-\frac{1}{2}x+1} \Rightarrow f'(x) = 2 \cdot -\frac{1}{2} \cdot e^{-\frac{1}{2}x+1} = -e^{-\frac{1}{2}x+1}.$

$$\int_0^4 \sqrt{1 + (-e^{-\frac{1}{2}x+1})^2} dx \text{ (fnInt)} \approx 6,392 \Rightarrow \text{omtrek} \approx 6,392 + 2e + 4 + 2e^{-1} \approx 16,56.$$

$\text{fnInt}(\sqrt{1+V_2^2}, X, 0, 4)$	
$V_1: 2 * e^{(-1/2X+1)}$	6.392397027
$V_2: -e^{(-1/2X+1)}$	16.56471957



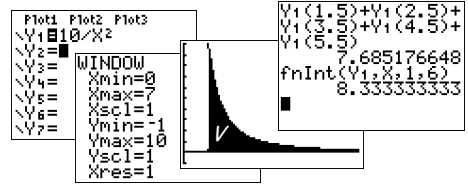
73c $I(L) = \pi \cdot 4^2 \cdot \frac{2}{e} + \int_{\frac{2}{e}}^{2e} \pi x^2 dy = \frac{32\pi}{e} + \int_{\frac{2}{e}}^{2e} \pi \cdot (2 - 2\ln(\frac{1}{2}y))^2 dy \text{ (fnInt)} \approx 81,16.$

$\text{fnInt}(\pi * 4^2 * V_1(4) + \text{fnInt}(\pi * V_2^2, X, 2/e, 2e))$	81.16083477
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Diagnostische toets

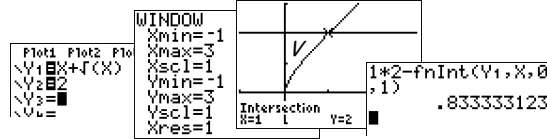
D1a \square De middens van de intervallen zijn 1,5, 2,5, 3,5, 4,5 en 5,5.
 $O(V) \approx f(1,5) \cdot 1 + f(2,5) \cdot 1 + f(3,5) \cdot 1 + f(4,5) \cdot 1 + f(5,5) \cdot 1 \approx 7,69$.

D1b \square $O(V) = \int_1^6 f(x) dx$ (fnInt) $\approx 8,3333$.



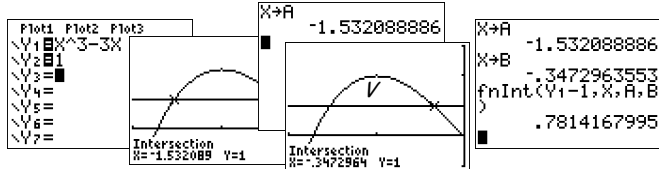
D2 \square $f(x) = 2$ (intersect) $\Rightarrow x = 1$.

$O(V) = 1 \cdot 2 - \int_0^1 f(x) dx$ (fnInt) $\approx 0,8333$.



D3 \square $f(x) = 1$ (intersect) $\Rightarrow x \approx -1,53 \vee x \approx -0,35$.

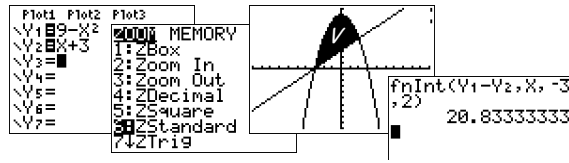
$O(V) = \int_{-1,53}^{-0,35} (f(x) - 1) dx$ (fnInt) $\approx 0,78$.



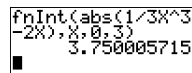
D4 \square $9 - x^2 = x + 3$ (intersect of)

$x^2 + x - 6 = 0 \Rightarrow (x+3)(x-2) = 0 \Rightarrow x = -3 \vee x = 2$.

$O(V) = \int_{-3}^2 (f(x) - g(x)) dx$ (fnInt) $\approx 20,83$.



D5 \square $O(V) = \int_0^3 |f(x)| dx$ (fnInt) $\approx 3,75$.

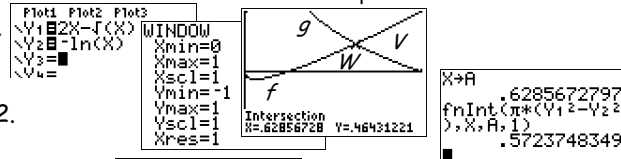


D6a \square $f(x) = 0 \Rightarrow 2x - \sqrt{x} = 0 \Rightarrow 2x = \sqrt{x} \Rightarrow 4x^2 = x \Rightarrow x = 0 \vee 4x = 1 \Rightarrow x = 0 \vee x = \frac{1}{4}$.

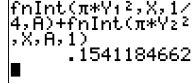
$g(x) = 0 \Rightarrow -\ln(x) = 0 \Rightarrow \ln(x) = 0 \Rightarrow x = e^0 = 1$.

$f(x) = g(x)$ (intersect) $\Rightarrow x \approx 0,63$.

$I = \int_{0,63}^1 \pi \cdot ((f(x))^2 - (g(x))^2) dx$ (fnInt) $\approx 0,572$.



D6b \square $I = \int_{0,25}^{0,63} \pi \cdot (f(x))^2 dx + \int_{0,63}^1 \pi \cdot (g(x))^2 dx$ (fnInt) $\approx 0,154$.



D7a \square $F(x) = (x^3 + 5)^4 + 2 \Rightarrow F'(x) = 4(x^3 + 5)^3 \cdot 3x^2 = 12x^2(x^3 + 5)^3$. Dus F is een primitieve van f .

D7b \square $G(x) = e^{x^3} - 3 \Rightarrow G'(x) = e^{x^3} \cdot 3x^2 = 3x^2 e^{x^3}$. Dus G is een primitieve van g .

D8a \square $f(x) = \frac{6}{x^4} = 6 \cdot x^{-4} \Rightarrow F(x) = 6 \cdot \frac{1}{-3} x^{-3} + c = -2x^{-3} + c = -\frac{2}{x^3} + c$.

D8b \square $f(x) = \frac{2x+6}{x^2} = \frac{2x}{x^2} + \frac{6}{x^2} = \frac{2}{x} + \frac{6}{x^2} = 2 \cdot \frac{1}{x} + 6 \cdot x^{-2} \Rightarrow F(x) = 2 \cdot \ln|x| + 6 \cdot \frac{1}{-1} x^{-1} + c = 2 \cdot \ln|x| - \frac{6}{x} + c$.

D8c \square $f(x) = 3 \cdot 2^x \Rightarrow F(x) = 3 \cdot \frac{1}{\ln(2)} \cdot 2^x + c = \frac{3 \cdot 2^x}{\ln(2)} + c$.

D8d \square $f(x) = 6e^x + x^2 \Rightarrow F(x) = 6 \cdot e^x + \frac{1}{3} x^3 + c$.

D8e \square $f(x) = \ln(x^2) = 2 \cdot \ln(x) \Rightarrow F(x) = 2 \cdot (x \ln(x) - x) + c = 2x \ln(x) - 2x + c$.

D8f \square $f(x) = {}^2\log(4 \cdot x) = {}^2\log(4) + {}^2\log(x) = {}^2\log(2^2) + \frac{\ln(x)}{\ln(2)} = 2 + \frac{\ln(x)}{\ln(2)} \Rightarrow F(x) = 2x + \frac{1}{\ln(2)} \cdot (x \ln(x) - x) + c$.

D9a \square $f(x) = \frac{x^2+1}{x} = \frac{x^2}{x} + \frac{1}{x} = x + \frac{1}{x} \Rightarrow F(x) = \frac{1}{2} x^2 + \ln|x| + c \Rightarrow e = \frac{1}{2} \cdot 1^2 + \ln(1) + c \Rightarrow e = \frac{1}{2} + 0 + c \Rightarrow e - \frac{1}{2} = c$.

Dus $F(x) = \frac{1}{2} x^2 + \ln|x| + e - \frac{1}{2}$.

D9b \square $O(V) = \int_1^2 \frac{x^2+1}{x} dx = \left[\frac{1}{2} x^2 + \ln|x| \right]_1^2 = (2 + \ln(2)) - \left(\frac{1}{2} - \ln(1) \right) = 1\frac{1}{2} + \ln(2)$.

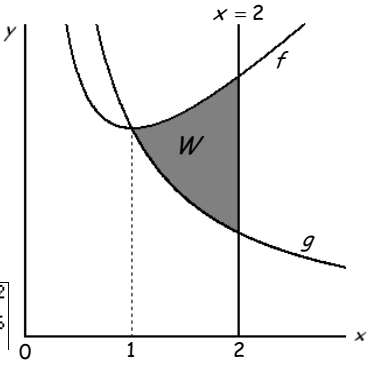
D9c $f(x) = g(x) \Rightarrow \frac{x^2+1}{x} = \frac{2}{x} \Rightarrow x^2+1=2 \Rightarrow x^2=1 \Rightarrow x=-1$ (vold. niet) $\vee x=1$ (voldoet).

$$I = \int_1^2 \pi \cdot \left(\frac{x^2+1}{x}\right)^2 dx - \int_1^2 \pi \cdot \left(\frac{2}{x}\right)^2 dx = \int_1^2 \pi \cdot \frac{x^4+2x^2+1}{x^2} dx - \int_1^2 \pi \cdot \frac{4}{x^2} dx$$

$$= \int_1^2 \pi \cdot \left(\frac{x^4}{x^2} + \frac{2x^2}{x^2} + \frac{1}{x^2}\right) dx - \int_1^2 \pi \cdot \frac{4}{x^2} dx = \int_1^2 \pi \cdot (x^2 + 2 + x^{-2}) dx - \int_1^2 \pi \cdot 4x^{-2} dx$$

$$= \left[\pi \cdot \left(\frac{1}{3}x^3 + 2x - x^{-1}\right) \right]_1^2 - \left[\pi \cdot (-4x^{-1}) \right]_1^2 = \left[\pi \cdot \left(\frac{1}{3}x^3 + 2x - \frac{1}{x}\right) \right]_1^2 + \left[\pi \cdot \frac{4}{x} \right]_1^2$$

$$= \pi \cdot \left(\frac{1}{3} \cdot 8 + 2 \cdot 2 - \frac{1}{2}\right) - \pi \cdot \left(\frac{1}{3} \cdot 1 + 2 \cdot 1 - \frac{1}{1}\right) + \pi \cdot \frac{4}{2} - \pi \cdot \frac{4}{1} = \frac{17}{6}\pi = 2\frac{5}{6}\pi.$$



D10a $f(x) = ((2x+6))^5 \Rightarrow F(x) = \frac{1}{2} \cdot \frac{1}{6} \cdot (2x+6)^6 + c = \frac{1}{12}(2x+6)^6 + c.$

D10b $f(x) = \frac{10}{(3x-1)^2} = 10 \cdot (3x-1)^{-2} \Rightarrow F(x) = \frac{1}{3} \cdot \frac{1}{-1} \cdot 10 \cdot (3x-1)^{-1} + c = \frac{-10}{3(3x-1)} + c.$

D10c $f(x) = (5x+2)^2 \cdot \sqrt{5x+2} = ((5x+2))^{2\frac{1}{2}} \Rightarrow F(x) = \frac{1}{5} \cdot \frac{1}{3\frac{1}{2}} \cdot (5x+2)^{3\frac{1}{2}} + c = \frac{2}{35} \cdot (5x+2)^3 \cdot \sqrt{5x+2} + c.$

D10d $f(x) = 4 \cdot e^{2x+3} \Rightarrow F(x) = \frac{1}{2} \cdot 4 \cdot e^{2x+3} + c = 2e^{2x+3} + c.$

D10e $f(x) = 8 \cdot 2^{2x-1} \Rightarrow F(x) = \frac{1}{2} \cdot 8 \cdot \frac{1}{\ln(2)} \cdot 2^{2x-1} + c = \frac{4}{\ln(2)} \cdot 2^{2x-1} + c.$

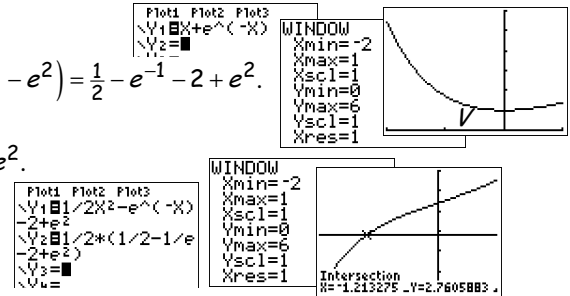
D10f $f(x) = \ln(2x+3) \Rightarrow F(x) = \frac{1}{2} \cdot ((2x+3) \cdot \ln(2x+3) - (2x+3)) + c = (x+1\frac{1}{2}) \cdot \ln(2x+3) - x - 1\frac{1}{2} + c.$

D11 $O(V) = \int_{-2}^1 (x + e^{-x}) dx = \left[\frac{1}{2}x^2 - e^{-x} \right]_{-2}^1 = \left(\frac{1}{2} - e^{-1}\right) - \left(\frac{1}{2} \cdot (-2)^2 - e^2\right) = \frac{1}{2} - e^{-1} - 2 + e^2.$

$O(\text{deel I}) = \int_{-2}^p (x + e^{-x}) dx = \left[\frac{1}{2}x^2 - e^{-x} \right]_{-2}^p = \frac{1}{2}p^2 - e^{-p} - 2 + e^2.$

$O(\text{deel I}) = \frac{1}{2}O(V) \Rightarrow \frac{1}{2}p^2 - e^{-p} - 2 + e^2 = \frac{1}{2} \cdot \left(\frac{1}{2} - e^{-1} - 2 + e^2\right).$

Intersect geeft dan $p \approx -1,213.$

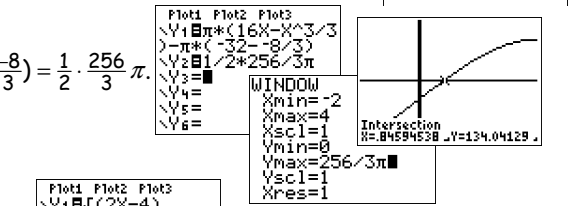


D12a $I(\text{bol}) = \frac{4}{3} \cdot \pi \cdot 4^3 = \frac{256}{3}\pi.$

D12b $I = \int_{-4}^{-2} \pi y^2 dx = \int_{-4}^{-2} \pi(16 - x^2) dx = \left[\pi(16x - \frac{1}{3}x^3) \right]_{-4}^{-2} = \pi(-32 - \frac{8}{3}) - \pi(-64 - \frac{64}{3}) = \frac{40}{3}\pi = 13\frac{1}{3}\pi.$

D12c $I(L_p) = \int_{-2}^p \pi y^2 dx = \left[\pi(16x - \frac{1}{3}x^3) \right]_{-2}^p = \pi(16p - \frac{p^3}{3}) - \pi(-32 - \frac{8}{3}) = \frac{1}{2} \cdot \frac{256}{3}\pi.$

Intersect geeft $p \approx 0,85.$

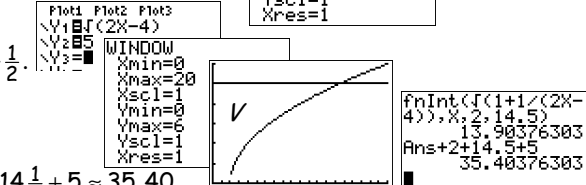


D13a $f(x) = 0 \Rightarrow \sqrt{2x-4} = 0 \Rightarrow 2x-4=0 \Rightarrow 2x=4 \Rightarrow x=2.$

$f(x) = 5 \Rightarrow \sqrt{2x-4} = 5 \Rightarrow 2x-4=25 \Rightarrow 2x=29 \Rightarrow x = \frac{29}{2} = 14\frac{1}{2}.$

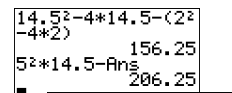
$f(x) = \sqrt{2x-4} \Rightarrow f'(x) = \frac{1}{2 \cdot \sqrt{2x-4}} \cdot 2 = \frac{1}{\sqrt{2x-4}}.$

$\int_2^{14\frac{1}{2}} \sqrt{1 + \left(\frac{1}{\sqrt{2x-4}}\right)^2} dx$ (fnInt) $\approx 13,903 \Rightarrow$ omtrek $= 2 + 13,903 + 14\frac{1}{2} + 5 \approx 35,40.$



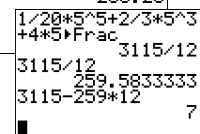
D13b $\int_2^{14\frac{1}{2}} \pi y^2 dx = \int_2^{14\frac{1}{2}} \pi \cdot (2x-4) dx = \left[\pi(x^2 - 4x) \right]_2^{14\frac{1}{2}} = \pi(14\frac{1}{2}^2 - 4 \cdot 14\frac{1}{2}) - \pi(2^2 - 4 \cdot 2) = 156\frac{1}{4}\pi.$

$I = \pi \cdot 5^2 \cdot 14\frac{1}{2} - 156\frac{1}{4}\pi = 206\frac{1}{4}\pi.$



D13c $\int_0^5 \pi x^2 dy = \int_0^5 \pi \cdot \left(\frac{1}{4}y^4 + 2y^2 + 4\right) dy = \left[\pi\left(\frac{1}{20}y^5 + \frac{2}{3}y^3 + 4y\right) \right]_0^5 = \pi\left(\frac{1}{20} \cdot 5^5 + \frac{2}{3} \cdot 5^3 + 4 \cdot 5\right) = 259\frac{7}{12}\pi.$

Gebruik: $y = \sqrt{2x-4} \Rightarrow 2x-4 = y^2 \Rightarrow 2x = y^2 + 4 \Rightarrow x = \frac{1}{2}y^2 + 2 \Rightarrow x^2 = \frac{1}{4}y^4 + 2y^2 + 4.$



Gemengde opgaven 10. Integraalrekening

G13a \square $f(x) = 7 \cdot \log(\sqrt{5x}) \Rightarrow F(x) = 7 \cdot \frac{1}{\ln(10)} \cdot \frac{1}{5} \cdot (5x \cdot \ln(5x) - 5x) + c = 7 \cdot \frac{x \cdot \ln(5x) - x}{\ln(10)} + c.$

G13b \square $f(x) = e^{\frac{1}{2}x-1} \Rightarrow F(x) = \frac{1}{\frac{1}{2}} \cdot e^{\frac{1}{2}x-1} + c = 2e^{\frac{1}{2}x-1} + c.$

G13c \square $f(x) = 5 \cdot \ln(2x-4)^2 = 10 \cdot \ln(\sqrt{2x-4}) \Rightarrow F(x) = \frac{1}{2} \cdot 10 \cdot ((2x-4)\ln(2x-4) - (2x-4)) + c = 5(2x-4)\ln(2x-4) - 5(2x-4) + c.$

G13d \square $f(x) = \ln(x^2 - 6x + 9) = \ln(x-3)^2 = 2 \cdot \ln(|x-3|) \Rightarrow F(x) = \frac{1}{1} \cdot 2 \cdot ((x-3)\ln(x-3) - (x-3)) + c = 2(x-3)\ln(x-3) - 2(x-3) + c.$

G13e \square $f(x) = \frac{x^4 - 6x^2 \cdot \sqrt{x} + 8x}{x^3} = x - 6x^{-\frac{1}{2}} + 8x^{-2} \Rightarrow F(x) = \frac{1}{2} \cdot x^2 - 6 \cdot \frac{1}{\frac{1}{2}} \cdot x^{\frac{1}{2}} + 8 \cdot \frac{1}{-1} \cdot x^{-1} + c = \frac{1}{2}x^2 - 12 \cdot \sqrt{x} - \frac{8}{x} + c.$

G13f \square $f(x) = (x^2 + 3)^2 = x^4 + 6x^2 + 9 \Rightarrow F(x) = \frac{1}{5} \cdot x^5 + 6 \cdot \frac{1}{3} \cdot x^3 + 9x + c = \frac{1}{5}x^5 + 2x^3 + 9x + c.$

G14a \square $f(x) = \sqrt{6x+3} = ((6x+3))^{\frac{1}{2}} \Rightarrow F(x) = \frac{1}{6} \cdot \frac{1}{\frac{3}{2}} \cdot (6x+3)^{\frac{3}{2}} + c = \frac{1}{6} \cdot \frac{2}{3} \cdot (6x+3)^{\frac{3}{2}} + c = \frac{1}{9} \cdot (6x+3) \cdot \sqrt{6x+3} + c.$

G14b \square $f(x) = \frac{10}{2x-1} = 10 \cdot \frac{1}{2x-1} \Rightarrow F(x) = 10 \cdot \frac{1}{2} \cdot \ln|2x-1| + c = 5 \cdot \ln|2x-1| + c.$

G14c \square $f(x) = ((3x-6))^{-2} \Rightarrow F(x) = \frac{1}{3} \cdot \frac{1}{-1} \cdot (3x-6)^{-1} + c = -\frac{1}{3} \cdot \frac{1}{3x-6} + c = \frac{1}{3(3x-6)} + c.$

G14d \square $f(x) = (2x+5)^{-1} = \frac{1}{2x+5} \Rightarrow F(x) = \frac{1}{2} \cdot \ln|2x+5| + c.$

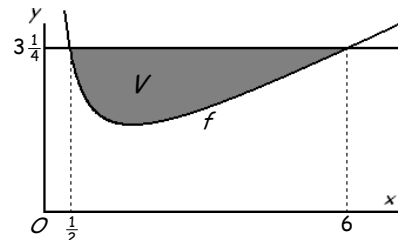
G14e \square $f(x) = 10^{\frac{2x-3}{2}} \Rightarrow F(x) = \frac{1}{2} \cdot \frac{1}{\ln(10)} \cdot 10^{2x-3} + c = \frac{10^{2x-3}}{2 \cdot \ln(10)} + c.$

G14f \square $f(x) = \frac{8^x-1}{2^x} = \frac{8^x}{2^x} - \frac{1}{2^x} = 4^x - 2^{-x} \Rightarrow F(x) = \frac{1}{\ln(4)} \cdot 4^x - \frac{1}{-1} \cdot \frac{1}{\ln(2)} \cdot 2^{-x} + c = \frac{4^x}{\ln(4)} + \frac{2^{-x}}{\ln(2)} + c.$

G15a \square $f(x) = 3\frac{1}{4} \Rightarrow \frac{x^2+3}{2x} = 3\frac{1}{4} \Rightarrow x^2+3 = 6\frac{1}{2}x \Rightarrow x^2 - 6\frac{1}{2}x + 3 = 0 \Rightarrow (x - \frac{1}{2})(x - 6) = 0 \Rightarrow x = \frac{1}{2} \vee x = 6.$

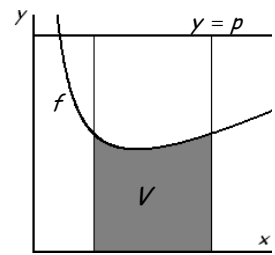
$O(V) = \int_{\frac{1}{2}}^6 (3\frac{1}{4} - \frac{x^2+3}{2x}) dx = \int_{\frac{1}{2}}^6 (3\frac{1}{4} - \frac{1}{2}x - \frac{3}{2} \cdot \frac{1}{x}) dx$
 $= [3\frac{1}{4}x - \frac{1}{4}x^2 - \frac{3}{2} \cdot \ln(x)]_{\frac{1}{2}}^6 = (19\frac{1}{2} - 9 - 1\frac{1}{2}\ln(6)) - (1\frac{5}{8} - \frac{1}{16} - 1\frac{1}{2}\ln(\frac{1}{2}))$
 $= 10\frac{1}{2} - 1\frac{5}{8} + \frac{1}{16} - 1\frac{1}{2}\ln(6) + 1\frac{1}{2}\ln(\frac{1}{2}) = 8\frac{15}{16} - 1\frac{1}{2}\ln(12).$

```
10+1/2-(1+5/8)+1
/16+frac 143/16
143/16 8.9375
Ans=8+frac 15/16
```



G15b \square $O(V) = \int_1^3 (\frac{x^2+3}{2x}) dx = [\frac{1}{4}x^2 + \frac{3}{2} \cdot \ln(x)]_1^3 = (2\frac{1}{4} + 1\frac{1}{2}\ln(3)) - (\frac{1}{4} + 1\frac{1}{2}\ln(1)) = 2 + 1\frac{1}{2}\ln(3).$

$O(V) = \frac{1}{2} \cdot O(\text{rechthoek}) \Rightarrow 2 + 1\frac{1}{2}\ln(3) = \frac{1}{2} \cdot 2 \cdot p \Rightarrow p = 2 + 1\frac{1}{2}\ln(3).$

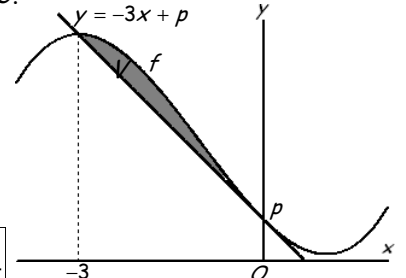


G16a \square $f_p(x) = \frac{1}{3}x^3 + x^2 - 3x + p \Rightarrow f_p'(x) = x^2 + 2x - 3.$ Stel de raaklijn is $y = ax + b.$
 $a = f_p'(0) = -3$ en $b = f_p(0) = p \Rightarrow y = -3x + p.$

$f_p(x) = -3x + p \Rightarrow \frac{1}{3}x^3 + x^2 - 3x + p = -3x + p \Rightarrow \frac{1}{3}x^3 + x^2 = 0 \Rightarrow x^2 \cdot (\frac{1}{3}x + 1) = 0 \Rightarrow x = 0 \vee \frac{1}{3}x = -1 \Rightarrow x = 0 \vee x = -3.$

$O(V) = \int_{-3}^0 (f_p(x) - (-3x + p)) dx = \int_{-3}^0 (\frac{1}{3}x^3 + x^2 - 3x + p + 3x - p) dx$
 $= \int_{-3}^0 (\frac{1}{3}x^3 + x^2) dx = [\frac{1}{12}x^4 + \frac{1}{3}x^3]_{-3}^0 = 0 - (\frac{81}{12} - 9) = 2\frac{1}{4}.$

```
0-(1/12*(-3)^4+1/3*(-3)^3)+frac 9/4
```



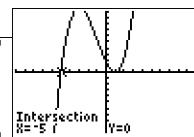
G16b \square $f_p'(x) = 0 \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow (x+3)(x-1) = 0 \Rightarrow x = -3 (< 0 \text{ vervalt}) \vee x = 1.$

$f_p(1) = 0 \Rightarrow \frac{1}{3} \cdot 1^3 + 1^2 - 3 \cdot 1 + p = 0 \Rightarrow p = 1\frac{2}{3}.$

$f_{1\frac{2}{3}}(x) = 0 \Rightarrow \frac{1}{3}x^3 + x^2 - 3x + 1\frac{2}{3} = 0$ (intersect) $\Rightarrow x = -5.$

$O(W) = \int_{-5}^1 (f_{1\frac{2}{3}}(x)) dx$ (fnInt) = 36.

```
Plot1 Plot2 Plot3
V1:1/3x^3+x^2-3x
+5/3
V2:0
V3:0
V4:0
V5:
V6:
V7:
2000 MEMORY
1:ZBox
2:Zoom In
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig
```

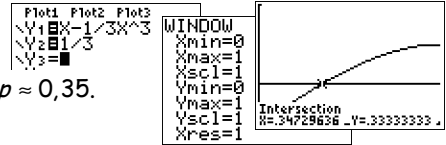


$$G17a \quad I = \int_{\frac{1}{3}r}^{\frac{1}{2}r} \pi y^2 dx = \int_{\frac{1}{3}r}^{\frac{1}{2}r} \pi(r^2 - x^2) dx = \left[\pi(r^2 x - \frac{1}{3} x^3) \right]_{\frac{1}{3}r}^{\frac{1}{2}r} = \pi(r^2 \cdot \frac{1}{2}r - \frac{1}{3} \cdot (\frac{1}{2}r)^3) - \pi(r^2 \cdot \frac{1}{3}r - \frac{1}{3} \cdot (\frac{1}{3}r)^3) = \frac{89}{649} \pi r^3.$$

$$G17b \quad I(L) = \int_{-pr}^{pr} \pi y^2 dx = 2 \cdot \int_0^{pr} \pi(r^2 - x^2) dx = 2 \cdot \left[\pi(r^2 x - \frac{1}{3} x^3) \right]_0^{pr} = 2\pi(r^2 \cdot pr - \frac{1}{3} \cdot (pr)^3) - 2\pi \cdot 0 = 2\pi r^3 \cdot (p - \frac{1}{3} p^3).$$

$$I(B) = \frac{4}{3} \pi r^3.$$

$$I(L) = \frac{1}{2} I(B) \Rightarrow 2\pi r^3 \cdot (p - \frac{1}{3} p^3) = \frac{2}{3} \pi r^3 \Rightarrow p - \frac{1}{3} p^3 = \frac{1}{3} \text{ (intersect)} \Rightarrow p \approx 0,35.$$



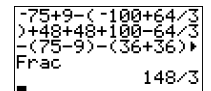
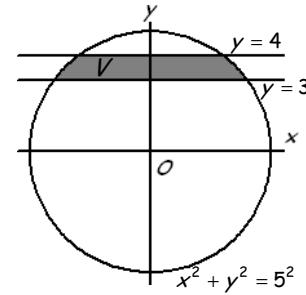
G18 $y = 3$ invullen in $x^2 + y^2 = 25$ geeft $x^2 + 9 = 25 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$.
 $y = 4$ invullen in $x^2 + y^2 = 25$ geeft $x^2 + 16 = 25 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$.

$$I(L) = \int_{-4}^{-3} \pi y^2 dx + \int_{-3}^3 \pi \cdot 4^2 dx + \int_3^4 \pi y^2 dx - \int_{-4}^{-3} \pi \cdot 3^2 dx$$

$$= \int_{-4}^{-3} \pi \cdot (25 - x^2) dx + \int_{-3}^3 16\pi dx + \int_3^4 \pi \cdot (25 - x^2) dx - \int_{-4}^{-3} 9\pi dx$$

$$= \left[\pi(25x - \frac{1}{3} x^3) \right]_{-4}^{-3} + [16\pi x]_{-3}^3 + \left[\pi(25x - \frac{1}{3} x^3) \right]_3^4 - [9\pi x]_{-4}^{-3}$$

$$= \pi(25 \cdot -3 - \frac{1}{3} \cdot (-3)^3) - \pi(25 \cdot -4 - \frac{1}{3} \cdot (-4)^3) + 48\pi - 48\pi + \pi(25 \cdot 4 - \frac{1}{3} \cdot 4^3) - \pi(25 \cdot 3 - \frac{1}{3} \cdot 3^3) - (36\pi - 36\pi) = \pi(-75 + 9) - \pi(-100 + \frac{64}{3}) + 48\pi + 48\pi + \pi(100 - \frac{64}{3}) - \pi(75 - 9) - (36\pi + 36\pi) = \frac{148}{3} \pi = 49 \frac{1}{3} \pi.$$



$$G19a \quad f_0(x) = 0 \Rightarrow -\frac{1}{3} x^3 + 4x = 0 \Rightarrow x^3 - 12x = 0 \Rightarrow x \cdot (x^2 - 12) = 0 \Rightarrow x = 0 \vee x^2 = 12 \Rightarrow x = 0 \vee x = \pm\sqrt{12} = \pm 2\sqrt{3}.$$

$$O(V) = \int_0^p \left(-\frac{1}{3} x^3 + 4x\right) dx = \left[-\frac{1}{12} x^4 + 2x^2\right]_0^p = \left(-\frac{1}{12} p^4 + 2p^2\right) - 0 = -\frac{1}{12} p^4 + 2p^2.$$

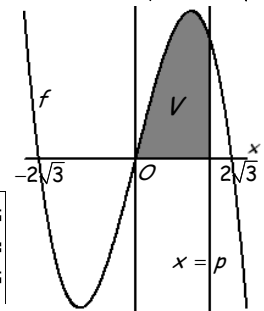
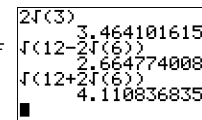
$$O(V) = 10 \Rightarrow -\frac{1}{12} p^4 + 2p^2 = 10 \text{ (stel } p^2 = t) \Rightarrow -\frac{1}{12} t^2 + 2t - 10 = 0 \Rightarrow t^2 - 24t + 120 = 0.$$

$$D = (24)^2 - 4 \cdot 1 \cdot 120 = 96 \Rightarrow \sqrt{D} = \sqrt{96} = \sqrt{16 \cdot 6} = 4 \cdot \sqrt{6}$$

$$t = p^2 = \frac{24 - 4 \cdot \sqrt{6}}{2} = 12 - 2\sqrt{6} \vee t = p^2 = \frac{24 + 4 \cdot \sqrt{6}}{2} = 12 + 2\sqrt{6}$$

$$p = \sqrt{12 - 2\sqrt{6}} \vee p = -\sqrt{12 - 2\sqrt{6}} \vee p = \sqrt{12 + 2\sqrt{6}} \vee p = -\sqrt{12 + 2\sqrt{6}}$$

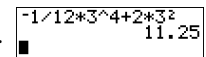
$$\text{Er moet gelden } 0 < p < 2\sqrt{3} = \sqrt{12} \Rightarrow \sqrt{12 - 2\sqrt{6}}.$$



$$G19b \quad \int_0^3 \left(-\frac{1}{3} x^3 + 4x + a\right) dx = \left[-\frac{1}{12} x^4 + 2x^2 + ax\right]_0^3 = \left(-\frac{1}{12} \cdot 3^4 + 2 \cdot 3^2 + a \cdot 3\right) - 0 = 3a + 11 \frac{1}{4}.$$

$$O(W) = 10 \Rightarrow 3a + 11 \frac{1}{4} = 10 \vee 3a + 11 \frac{1}{4} = -10 \text{ (grafiek onder de } x\text{-as)}$$

$$3a = -1 \frac{1}{4} = -\frac{5}{4} \vee 3a = -21 \frac{1}{4} \Rightarrow a = -\frac{5}{12} \text{ (voldoet niet, want er ontstaat niet HET vlakdeel } W) \vee a = -7 \frac{1}{12} \text{ (deze voldoet).}$$



$$G20a \quad f(x) = \frac{1}{4} x^2 \Rightarrow f'(x) = \frac{1}{2} x.$$

$$f'(x) = 1 \Rightarrow \frac{1}{2} x = 1 \Rightarrow x = 2. \text{ Dus raakpunt } R(2, f(2)) = R(2, 1).$$

Raaklijn door $R(2, 1)$ van de vorm $y = x + b$ met $1 = 2 + b \Rightarrow b = -1$. Dus de raaklijn door $R(2, 1)$ is $y = x - 1$.

$$g(x) = -\frac{4}{x^2} = -4x^{-2} \Rightarrow g'(x) = 8x^{-3} = \frac{8}{x^3}.$$

$$g'(x) = 1 \Rightarrow \frac{8}{x^3} = 1 \Rightarrow x^3 = 8 \Rightarrow x = 2. \text{ Dus raakpunt } S(2, g(2)) = S(2, -1).$$

Raaklijn door $S(2, -1)$ van de vorm $y = x + b$ met $-1 = 2 + b \Rightarrow b = -3$. Dus de raaklijn door $S(2, -1)$ is $y = x - 3$.

De raaklijnen snijden de y -as in $(0, -1)$ en $(0, -3)$. De diagonaal van het vierkant is dus 2.

$$G20b \quad f(a) = \frac{1}{4} a^2 \Rightarrow C(a, \frac{1}{4} a^2) \text{ en } D(-a, \frac{1}{4} a^2); g(a) = -\frac{4}{a^2} \Rightarrow B(a, -\frac{4}{a^2}) \text{ en } A(-a, -\frac{4}{a^2}).$$

$$O(ABCD) = 2a \cdot \left(\frac{1}{4} a^2 + \frac{4}{a^2}\right) = \frac{1}{2} a^3 + \frac{8}{a}.$$

$$O(\text{vlakdeel boven de grafiek van } f) = 2 \cdot \int_0^a \left(\frac{1}{4} a^2 - \frac{1}{4} x^2\right) dx = 2 \cdot \left[\frac{1}{4} a^2 x - \frac{1}{12} x^3\right]_0^a = 2 \cdot \left(\frac{1}{4} a^3 - \frac{1}{12} a^3\right) = \frac{1}{3} a^3.$$

$$O(\text{vlakdeel boven de grafiek van } f) = \frac{1}{2} \cdot O(ABCD)$$

$$\frac{1}{3} a^3 = \frac{1}{4} a^3 + \frac{4}{a} \Rightarrow \frac{1}{12} a^3 = \frac{4}{a} \Rightarrow a^4 = 48 \Rightarrow a = \sqrt[4]{48}.$$

G21a \square $A(0, e^0) = A(0, 1)$ en $B(1, e^{-1}) = B(1, \frac{1}{e})$.

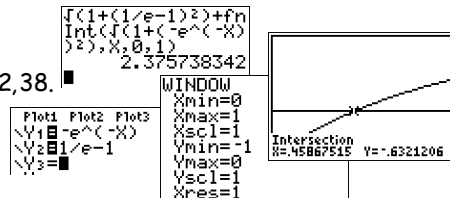
$AB: y = (\frac{1}{e} - 1)x + b$ door $A(0, 1) \Rightarrow b = 1$. Dus $AB: y = (\frac{1}{e} - 1)x + 1$.

$$O(V) = \int_0^1 \left((\frac{1}{e} - 1)x + 1 \right) dx - \int_0^1 e^{-x} dx = \left[\frac{1}{2} \cdot (\frac{1}{e} - 1)x^2 + x \right]_0^1 - \left[-e^{-x} \right]_0^1$$

$$= \frac{1}{2} \cdot (\frac{1}{e} - 1) \cdot 1^2 + 1 - 0 - (-e^{-1} - -e^0) = \frac{1}{2e} - \frac{1}{2} + 1 + e^{-1} - 1 = \frac{1}{2e} - \frac{1}{2} + \frac{1}{e} = \frac{3}{2e} - \frac{1}{2}$$

G21b \square $f(x) = e^{-x} \Rightarrow f'(x) = -e^{-x}$.

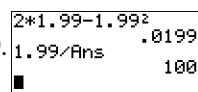
Omtrek = AB + boog $AB = \sqrt{1^2 + (\frac{1}{e} - 1)^2} + \int_0^1 \sqrt{1 + (-e^{-x})^2} dx$ (fnInt) $\approx 2,38$.



G21c \square $f'(x) = rc_{AB} \Rightarrow -e^{-x} = \frac{1}{e} - 1$ (intersect) $\Rightarrow x \approx 0,46$.

G22a \square $O = \int_0^2 3(2x - x^2) dx - \int_0^2 2(2x - x^2) dx = \left[3(x^2 - \frac{1}{3}x^3) \right]_0^2 - \left[2(x^2 - \frac{1}{3}x^3) \right]_0^2 = 3(4 - \frac{8}{3}) - 2(4 - \frac{8}{3}) = 4 - \frac{8}{3} = 1\frac{1}{3}$.

G22b \square $x = 1,99$ invullen in $n(2x - x^2) = x$ geeft $n \cdot (2 \cdot 1,99 - 1,99^2) = 1,99 \Rightarrow n \cdot 0,0199 = 1,99 \Rightarrow n = 100$.
Dus voor $n > 100$ is $x_{S_n} > 1,99$.



G22c \square $y = n(2x - x^2) = 2nx - nx^2 \Rightarrow y' = \left[\frac{dy}{dx} \right] = 2n - 2nx$. Dus $\left[\frac{dy}{dx} \right]_{x=0} = 2n \Rightarrow$ raaklijn in O is $y = 2nx$.

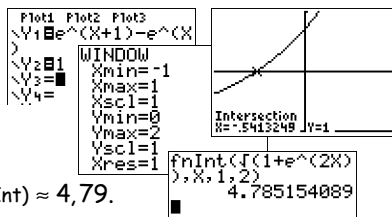
$x = 1$ invullen in $y = 2nx$ geeft $y = 2n \Rightarrow R_n(1, 2n)$
 $x = 1$ invullen in $y = n(2x - x^2)$ geeft $y = n \cdot (2 \cdot 1 - 1^2) = n \cdot 1 = n \Rightarrow T_n(1, n)$ } $\Rightarrow T_n$ is het midden van AR_n .

G23a \square $O = (a+1-a) \cdot e^{a+1} - \int_a^{a+1} e^x dx = e^{a+1} - [e^x]_a^{a+1} = e^{a+1} - (e^{a+1} - e^a) = e^{a+1} - e^{a+1} + e^a = e^a$.

$O = 3 \Rightarrow e^a = 3 \Rightarrow a = \ln(3)$.

G23b \square $A(a, e^a)$ en $B(a+1, e^{a+1}) \Rightarrow rc_{AB} = \frac{e^{a+1} - e^a}{a+1-a} = e^{a+1} - e^a$.

$rc_{AB} < 1 \Rightarrow e^{a+1} - e^a < 1$ (intersect) $\Rightarrow a < -0,54$.



G23c \square $f(x) = e^x \Rightarrow f'(x) = e^x$. Boog $AB = \int_1^2 \sqrt{1 + (e^x)^2} dx$ (fnInt) $\approx 4,79$.

G23d \square $OPAQ$ wentelen om de x -as geeft: $I_1 = \int_0^1 \pi e^2 dx = [\pi e^2 x]_0^1 = \pi e^2$ of $I_1 = G \cdot h = \pi \cdot e^2 \cdot 1 = \pi e^2$.

Het deel onder de grafiek van f wentelen om de x -as geeft:

$$I_2 = \int_0^1 \pi (e^x)^2 dx = \int_0^1 \pi \cdot e^{2x} dx = \left[\frac{1}{2} \pi e^{2x} \right]_0^1 = \frac{1}{2} \pi e^2 - \frac{1}{2} \pi$$

Het deel boven de grafiek van f wentelen om de x -as geeft: $I_3 = I_1 - I_2$.

Het verschil tussen de inhouds is: $I_3 - I_2 = (I_1 - I_2) - I_2 = I_1 - 2I_2 = \pi e^2 - 2 \cdot (\frac{1}{2} \pi e^2 - \frac{1}{2} \pi) = \pi$.

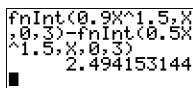
G24a \square $O = \int_0^3 0,9x^{1\frac{1}{2}} dx - \int_0^3 0,5x^{1\frac{1}{2}} dx$ (fnInt) $\approx 2,49$.

G24b \square $I = \int_0^3 \pi \cdot (2x^{1\frac{1}{2}})^2 dx = \int_0^3 4\pi x^3 dx = \left[\frac{1}{4} \cdot 4\pi x^4 \right]_0^3 = [\pi x^4]_0^3 = 81\pi$.

G24c \square $f_2(x) = \frac{2}{3} \cdot x^{1\frac{1}{2}} \Rightarrow f_2'(x) = \frac{2}{3} \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} = \sqrt{x}$.

$$L(\frac{2}{3}) = \int_0^3 \sqrt{1 + (\sqrt{x})^2} dx = \int_0^3 \sqrt{1+x} dx = \int_0^3 ((1+x))^{\frac{1}{2}} dx$$

$$= \left[\frac{1}{1\frac{1}{2}} (1+x)^{1\frac{1}{2}} \right]_0^3 = \left[\frac{2}{3} (1+x)^{1\frac{1}{2}} \right]_0^3 = \left[\frac{2}{3} \cdot (1+x) \cdot \sqrt{(1+x)} \right]_0^3 = \frac{2}{3} \cdot 4 \cdot \sqrt{4} - \frac{2}{3} \cdot 1 \cdot \sqrt{1} = \frac{16}{3} - \frac{2}{3} = \frac{14}{3}$$



TI-84 10. Integreren

1a $\int_{-3}^1 (x^2 + 4) dx$ (fnInt) = $25\frac{1}{3}$.

```

4: fnInt(
5: *f(
6: fMin(
7: fMax(
8: fnDeriv(
9: fnInt(
0: Solver...
    
```

1b $\int_{-2}^2 (4 - x^2) dx$ (fnInt) = $10\frac{2}{3}$.

```

fnInt(4-X^2,X,-2,2)
Ans-10*Frac
2/3
    
```

1c $\int_0^4 (\frac{1}{2}x^3 + 1) dx$ (fnInt) = 36.

```

fnInt(1/2X^3+1,X,0,4)
36
    
```

1d $\int_{-2}^3 (\frac{1}{4}x^4 + x + 3) dx$ (fnInt) = $31\frac{1}{4}$.

```

fnInt(1/4X^4+X+3,X,-2,3)
31.25
    
```

2a $\int_1^5 \frac{1}{x} dx$ (fnInt) $\approx 1,61$.

```

fnInt(1/X,X,1,5)
1.609437912
fnInt(4X/(X^2+1),X,0,4)
5.666426688
    
```

2c $\int_0^2 2^x dx$ (fnInt) $\approx 4,33$.

```

fnInt(2^X,X,0,2)
4.328085123
fnInt(e^(X)/(e^(X)+1),X,1,2)
.8136663235
    
```

2b $\int_0^4 \frac{4x}{x^2+1} dx$ (fnInt) $\approx 5,67$.

2d $\int_1^2 \frac{e^x}{e^x+1} dx$ (fnInt) $\approx 0,81$.

TI-84 11. Integralen

1 $f(x) = g(x)$ (intersect) \Rightarrow
 $x_A \approx -3,031$ en $x_B \approx 2,547$.

De grafiek van $f(x)$ loopt tussen de snijpunten boven de grafiek van $g(x) \Rightarrow$

$O(V) = \int_{x_A}^{x_B} (f(x) - g(x)) dx$ (fnInt) $\approx 42,48$.

$f(x)$	primitieven $F(x)$
ax^n	$\frac{a}{n+1} \cdot x^{n+1} + c$
g^x	$\frac{1}{\ln(g)} \cdot g^x + c$
e^x	$e^x + c$
$\frac{1}{x}$	$\ln x + c$
$\ln(x)$	$x \cdot \ln(x) - x + c$
$g \log(x)$	$\frac{1}{\ln(g)} \cdot (x \cdot \ln(x) - x) + c$
$f(ax + b)$	$\frac{1}{a} \cdot F(ax + b) + c$

$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$.

- $O(\text{cirkel}) = \pi r^2$
- $I(\text{cilinder}) = Gh = \pi r^2 h$
- $I(\text{kegel}) = \frac{1}{3} Gh = \frac{1}{3} \pi r^2 h$
- $I(\text{bol}) = \frac{4}{3} \pi r^3$

De lengte van de grafiek van f tussen $x = a$ en $x = b$ is $\int_a^b \sqrt{1 + (f'(x))^2} dx$.

Het vlakdeel V ligt rechts van de y -as en wordt ingesloten door de grafiek van de functie f , de y -as en de lijnen $y = a$ en $y = b$.

De inhoud van het lichaam L dat ontstaat als V om de y -as wentelt is $I(L) = \int_a^b \pi x^2 dy$.

